EXERCISES FOR SECTION 4.4

Think About It **In Exercises 1–4, use a graphing utility to graph the integrand. Use the graph to determine whether the definite integral is positive, negative, or zero.**

1. $\displaystyle\int_0^{\pi} \frac{4}{x^2+1}\,dx$
2. $\displaystyle\int_0^{\pi} \cos x\,dx$
3. $\displaystyle\int_{-2}^{2} x\sqrt{x^2+1}\,dx$
4. $\displaystyle\int_{-2}^{2} x\sqrt{2-x}\,dx$

In Exercises 5–24, evaluate the definite integral of the algebraic function. Use a graphing utility to verify your result.

5. $\displaystyle\int_0^1 2x\,dx$
6. $\displaystyle\int_2^7 3\,dv$
7. $\displaystyle\int_{-1}^0 (x-2)\,dx$
8. $\displaystyle\int_2^5 (-3v+4)\,dv$
9. $\displaystyle\int_{-1}^1 (t^2-2)\,dt$
10. $\displaystyle\int_0^3 (3x^2+x-2)\,dx$
11. $\displaystyle\int_0^1 (2t-1)^2\,dt$
12. $\displaystyle\int_{-1}^1 (t^3-9t)\,dt$
13. $\displaystyle\int_1^2 \left(\frac{3}{x^2}-1\right)dx$
14. $\displaystyle\int_{-2}^{-1} \left(u-\frac{1}{u^2}\right)du$
15. $\displaystyle\int_1^4 \frac{u-2}{\sqrt{u}}\,du$
16. $\displaystyle\int_{-3}^3 v^{1/3}\,dv$
17. $\displaystyle\int_{-1}^1 (\sqrt[3]{t}-2)\,dt$
18. $\displaystyle\int_1^8 \sqrt{\frac{2}{x}}\,dx$
19. $\displaystyle\int_0^1 \frac{x-\sqrt{x}}{3}\,dx$
20. $\displaystyle\int_0^2 (2-t)\sqrt{t}\,dt$
21. $\displaystyle\int_{-1}^0 \left(t^{1/3}-t^{2/3}\right)dt$
22. $\displaystyle\int_{-8}^{-1} \frac{x-x^2}{2\sqrt[3]{x}}\,dx$
23. $\displaystyle\int_0^3 |2x-3|\,dx$
24. $\displaystyle\int_0^4 |x^2-4x+3|\,dx$

In Exercises 25–30, evaluate the definite integral of the trigonometric function. Use a graphing utility to verify your result.

25. $\displaystyle\int_0^{\pi} (1+\sin x)\,dx$
26. $\displaystyle\int_0^{\pi/4} \frac{1-\sin^2\theta}{\cos^2\theta}\,d\theta$
27. $\displaystyle\int_{-\pi/6}^{\pi/6} \sec^2 x\,dx$
28. $\displaystyle\int_{\pi/4}^{\pi/2} (2-\csc^2 x)\,dx$
29. $\displaystyle\int_{-\pi/3}^{\pi/3} 4\sec\theta\tan\theta\,d\theta$
30. $\displaystyle\int_{-\pi/2}^{\pi/2} (2t+\cos t)\,dt$

31. ***Depreciation*** A company purchases a new machine for which the rate of depreciation is $dV/dt = 10{,}000(t-6)$, $0 \le t \le 5$, where V is the value of the machine after t years. Set up and evaluate the definite integral that yields the total loss of value of the machine over the first 3 years.

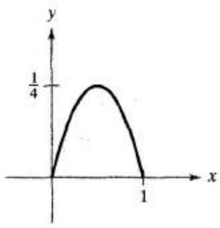
32. ***Buffon's Needle Experiment*** A horizontal plane is ruled with parallel lines 2 inches apart. If a 2-inch needle is tossed randomly onto the plane, the probability that the needle will touch a line is

$$P = \frac{2}{\pi}\int_0^{\pi/2} \sin\theta\,d\theta$$

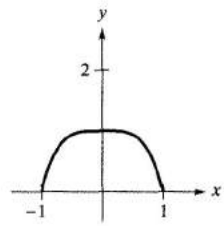
where θ is the acute angle between the needle and any one of the parallel lines. Find this probability.

In Exercises 33–38, determine the area of the indicated region.

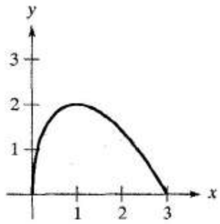
33. $y = x - x^2$



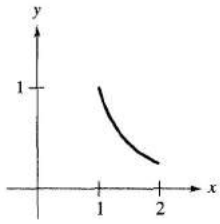
34. $y = 1 - x^4$



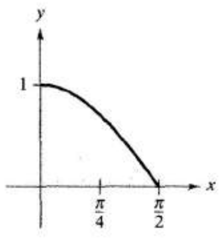
35. $y = (3-x)\sqrt{x}$



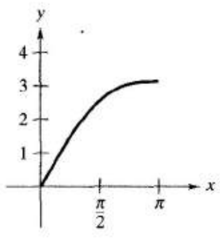
36. $y = \dfrac{1}{x^2}$



37. $y = \cos x$



38. $y = x + \sin x$



In Exercises 39–42, find the area of the region bounded by the graphs of the equations.

39. $y = 3x^2 + 1,\quad x = 0,\quad x = 2,\quad y = 0$
40. $y = 1 + \sqrt{x},\quad x = 0,\quad x = 4,\quad y = 0$
41. $y = x^3 + x,\quad x = 2,\quad y = 0$
42. $y = -x^2 + 3x,\quad y = 0$

In Exercises 43–46, find the value of c guaranteed by the *Mean Value Theorem for Integrals* for the function over the indicated interval.

Function	*Interval*
43. $f(x) = x - 2\sqrt{x}$	$[0, 2]$
44. $f(x) = \dfrac{9}{x^3}$	$[1, 3]$
45. $f(x) = 2\sec^2 x$	$[-\pi/4, \pi/4]$
46. $f(x) = \cos x$	$[-\pi/3, \pi/3]$

In Exercises 47–50, use a graphing utility to graph the function over the indicated interval. Find the average value of the function over the interval and all values of x in the interval for which the function equals its average value.

Function	*Interval*
47. $f(x) = 4 - x^2$	$[-2, 2]$
48. $f(x) = \dfrac{x^2 + 1}{x^2}$	$\left[\frac{1}{2}, 2\right]$
49. $f(x) = \sin x$	$[0, \pi]$
50. $f(x) = \cos x$	$[0, \pi/2]$

Think About It **In Exercises 51–56, use the graph of f shown in the figure. The shaded region A has an area of 1.5, and $\int_0^6 f(x)\,dx = 3.5$. Use this information to fill in the blanks.**

51. $\displaystyle\int_0^2 f(x)\,dx =$ **52.** $\displaystyle\int_2^6 f(x)\,dx =$

53. $\displaystyle\int_0^6 |f(x)|\,dx =$ **54.** $\displaystyle\int_0^2 -2f(x)\,dx =$

55. $\displaystyle\int_0^6 [2 + f(x)]\,dx =$

56. The average value of f over the interval $[0, 6]$ is .

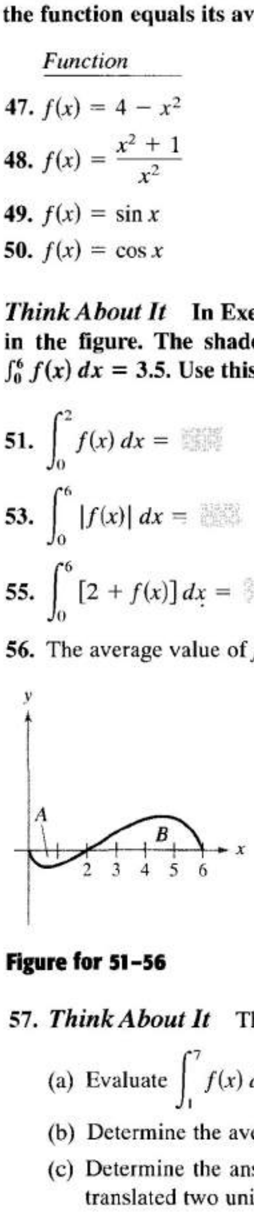


Figure for 51–56

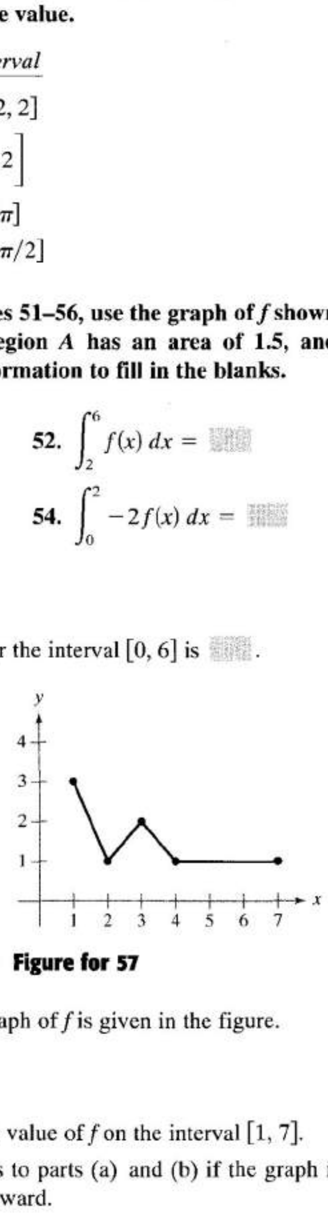


Figure for 57

57. ***Think About It*** The graph of f is given in the figure.

(a) Evaluate $\displaystyle\int_1^7 f(x)\,dx$.

(b) Determine the average value of f on the interval $[1, 7]$.

(c) Determine the answers to parts (a) and (b) if the graph is translated two units upward.

58. ***Average Profit*** A company introduces a new product, and the profit in thousands of dollars over the first 6 months is approximated by the model

$$P = 5(\sqrt{t} + 30), \qquad t = 1, 2, 3, 4, 5, 6.$$

(a) Use the model to complete the table and use the entries to calculate (arithmetically) the average profit over the first 6 months.

t	1	2	3	4	5	6
P						

(b) Find the average value of the profit function by integration and compare the result with that in part (a). (Integrate over the interval $[0.5, 6.5]$.)

(c) What, if any, is the advantage of using the approximation of the average given by the definite integral? (Note that the integral approximation utilizes all real values of t in the interval rather than just integers.)

59. ***Modeling Data*** A life insurance company needs a model to approximate the death rate of citizens during the years they are in the work force. The table gives the death rate R per 1000 for individuals of age x. *(Source: Department of Health and Human Services)*

x	20	30	40	50	60
R	1.0	1.4	2.3	4.7	11.7

A model for these data is

$$R = -91.1 - 6.313x + 0.035x^2 + 45.794\sqrt{x},$$
$$20 \le x \le 60.$$

(a) Use a graphing utility to plot the data and graph the model.

(b) Find the rate of increase of the death rate when $x = 40$ and $x = 50$.

(c) Find the average death rate for people between the ages of 30 and 40 and for people between the ages of 50 and 60.

60. ***Blood Flow*** The velocity v of the flow of blood at a distance r from the central axis of an artery of radius R is $v = k(R^2 - r^2)$, where k is the constant of proportionality. Find the average rate of flow of blood along a radius of the artery. (Use 0 and R as the limits of integration.)

61. ***Force*** The force F (in newtons) of a hydraulic cylinder in a press is proportional to the square of $\sec x$, where x is the distance (in meters) that the cylinder is extended in its cycle. The domain of F is $[0, \pi/3]$, and $F(0) = 500$.

(a) Find F as a function of x.

(b) Find the average force exerted by the press over the interval $[0, \pi/3]$.

62. ***Respiratory Cycle*** The volume V in liters of air in the lungs during a 5-second respiratory cycle is approximated by the model $V = 0.1729t + 0.1522t^2 - 0.0374t^3$ where t is the time in seconds. Approximate the average volume of air in the lungs during one cycle.

63. ***Modeling Data*** A department store manager wants to estimate the number of customers that enter the store from noon until closing at 9 P.M. The table shows the number of customers N entering the store during a randomly selected minute each hour from $t-1$ to t, with $t=0$ corresponding to noon.

t	1	2	3	4	5	6	7	8	9
N	6	7	9	12	15	14	11	7	2

(a) Draw a histogram of the data.

(b) Estimate the total number of customers entering the store between noon and 9 P.M.

(c) Use the regression capabilities of a graphing utility to find a model of the form

$N(t) = at^3 + bt^2 + ct + d$

for the data.

(d) Use a graphing utility to plot the data and graph the model.

(e) Use a graphing utility to evaluate $\int_0^9 N(t)\,dt$, and use the result to estimate the number of customers entering the store between noon and 9 P.M. Compare this with your answer in part (b).

(f) Estimate the average number of customers entering the store per minute between 3 P.M. and 7 P.M.

64. ***Modeling Data*** In the manufacturing process of a product, there is a repetitive heating cycle of 4 minutes. During a review of the process, the flow R (cubic feet per minute) of natural gas was measured in 1-minute intervals and the results were recorded in the table.

t	0	1	2	3	4
R	0	62	76	38	0

(a) Use a graphing utility to find a model of the form $R = at^4 + bt^3 + ct^2 + dt + e$ for the data.

(b) Use a graphing utility to plot the data and graph the model.

(c) Use the Fundamental Theorem of Calculus to approximate the number of cubic feet of natural gas used in one heating cycle.

65. ***Modeling Data*** A radio-controlled experimental vehicle is tested on a straight track. It starts from rest, and its velocity v (meters per second) is recorded in the table every 10 seconds for 1 minute.

t	0	10	20	30	40	50	60
v	0	5	21	40	62	78	83

(a) Use a graphing utility to find a model of the form $v = at^3 + bt^2 + ct + d$ for the data.

(b) Use a graphing utility to plot the data and graph the model.

(c) Use the Fundamental Theorem of Calculus to approximate the distance traveled by the vehicle during the test.

66. Use the function f in the figure and the function g defined by

$$g(x) = \int_0^x f(t)\,dt.$$

(a) Complete the table.

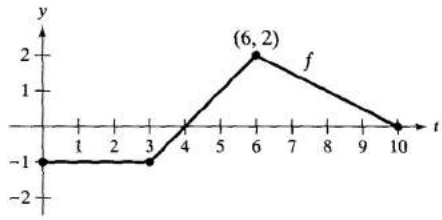
x	0	1	2	3	4	5	6	7	8	9	10
$g(x)$											

(b) Plot the points from the table in part (a).

(c) Where does g have its minimum? Explain.

(d) Which four consecutive points are collinear? Explain.

(e) Between which two consecutive points does g increase at the greatest rate? Explain.



In Exercises 67–72, (a) integrate to find F as a function of x and (b) demonstrate the Second Fundamental Theorem of Calculus by differentiating the result in part (a).

67. $F(x) = \int_0^x (t+2)\,dt$

68. $F(x) = \int_0^x t(t^2+1)\,dt$

69. $F(x) = \int_8^x \sqrt[3]{t}\,dt$

70. $F(x) = \int_4^x \sqrt{t}\,dt$

71. $F(x) = \int_{\pi/4}^x \sec^2 t\,dt$

72. $F(x) = \int_{\pi/3}^x \sec t \tan t\,dt$

In Exercises 73–78, use the Second Fundamental Theorem of Calculus to find $F'(x)$.

73. $F(x) = \int_{-2}^x (t^2 - 2t)\,dt$

74. $F(x) = \int_1^x \sqrt[4]{t}\,dt$

75. $F(x) = \int_{-1}^x \sqrt{t^4+1}\,dt$

76. $F(x) = \int_0^x \tan^4 t\,dt$

77. $F(x) = \int_0^x t\cos t\,dt$

78. $F(x) = \int_1^x \frac{t^2}{t^2+1}\,dt$

In Exercises 79–84, find $F'(x)$.

79. $F(x) = \int_x^{x+2} (4t+1)\,dt$

80. $F(x) = \int_{-x}^x t^3\,dt$

81. $F(x) = \int_0^{\sin x} \sqrt{t}\,dt$

82. $F(x) = \int_2^{x^2} \frac{1}{t^2}\,dt$

83. $F(x) = \int_0^{x^3} \sin t^2\,dt$

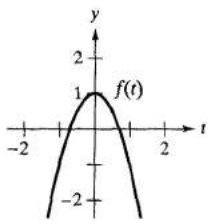
84. $F(x) = \int_0^{3x} \sqrt{1+t^3}\,dt$

In Exercises 85 and 86, sketch a graph of the function

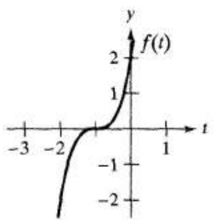
$$F(x) = \int_0^x f(t)\,dt.$$

State any relationship that may exist between the extrema and inflection points on the graphs of f and F.

85.



86.



87. *Cost* The total cost of purchasing and maintaining a piece of equipment for x years is

$$C(x) = 5000\left(25 + 3\int_0^x t^{1/4}\,dt\right).$$

(a) Perform the integration to write C as a function of x.

(b) Find $C(1)$, $C(5)$, and $C(10)$.

88. *Area* The area A between the graph of the function $g(t) = 4 - 4/t^2$ and the t-axis over the interval $[1, x]$ is

$$A(x) = \int_1^x \left(4 - \frac{4}{t^2}\right) dt.$$

(a) Find the horizontal asymptote of the graph of g.

(b) Integrate to find A as a function of x. Does the graph of A have a horizontal asymptote? Explain.

True or False? **In Exercises 89–91, determine whether the statement is true or false. If it is false, explain why or give an example that shows it is false.**

89. If $F'(x) = G'(x)$ on the interval $[a, b]$, then $F(b) - F(a) = G(b) - G(a)$.

90. If f is continuous on $[a, b]$, then f is integrable on $[a, b]$.

91. $\int_{-1}^{1} x^{-2}\,dx = \left[-x^{-1}\right]_{-1}^{1} = (-1) - 1 = -2$

92. Prove: $\frac{d}{dx}\left[\int_{u(x)}^{v(x)} f(t)\,dt\right] = f(v(x))v'(x) - f(u(x))u'(x)$.

93. Show that the function

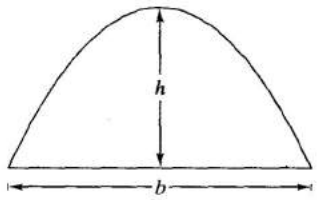
$$f(x) = \int_0^{1/x} \frac{1}{t^2+1}\,dt + \int_0^x \frac{1}{t^2+1}\,dt$$

is constant for $x > 0$.

94. Let $G(x) = \int_0^x \left[\int_0^s f(t)dt\right] ds$, where f is continuous for all real t. Find

(a) $G(0)$. (b) $G'(0)$. (c) $G''(x)$. (d) $G''(0)$.

95. *Area* Archimedes showed that the area of a parabolic arch is equal to $\frac{2}{3}$ the product of the base and the height (see figure).



(a) Graph the parabolic arch bounded by $y = 9 - x^2$ and the x-axis. Use an appropriate definite integral to find the area A.

(b) Find the base and height of the arch in part (a) and verify that $A = \frac{2}{3}bh$.

(c) Verify Archimedes's formula for the parabolic arch bounded by $y = 5x - x^2$ and the x-axis.

Rectilinear Motion **In Exercises 96–98, consider a particle moving along the x-axis where $x(t)$ is the position of the particle at time t, $x'(t)$ is its velocity, and $\int_a^b |x'(t)|\,dt$ is the distance the particle travels in the interval of time.**

96. The position function is $x(t) = t^3 - 6t^2 + 9t - 2$, $0 \le t \le 5$. Find the total distance the particle travels in 5 units of time.

97. Repeat Exercise 96 for the position function given by $x(t) = (t-1)(t-3)^2$, $0 \le t \le 5$.

98. A particle moves along the x-axis with velocity $v(t) = 1/\sqrt{t}$, $t > 0$. At time $t = 1$, its position is $x = 4$. Find the total distance traveled by the particle on the interval $1 \le t \le 4$.

SECTION PROJECT

Use a graphing utility to graph the function $y_1 = \sin^2 t$ on the interval $0 \le t \le \pi$. Let $F(x)$ be the following function of x.

$$F(x) = \int_0^x \sin^2 t\,dt$$

(a) Complete the table and explain why the values of F are increasing.

x	0	$\pi/6$	$\pi/3$	$\pi/2$	$2\pi/3$	$5\pi/6$	π
$F(x)$							

(b) Use the integration capabilities of a graphing utility to graph F.

(c) Use the differentiation capabilities of a graphing utility to graph $F'(x)$. How is this graph related to the graph in part (b)?

(d) Verify that the derivative of $y = (1/2)t - (\sin 2t)/4$ is $\sin^2 t$. Graph y and write a short paragraph about how this graph is related to those in parts (b) and (c).