EXERCISES FOR SECTION 4.5

In Exercises 1–6, complete the table by identifying u and du for the integral.

$\int f(g(x))g'(x)\,dx$	$u = g(x)$	$du = g'(x)\,dx$
1. $\int (5x^2+1)^2(10x)\,dx$		
2. $\int x^2\sqrt{x^3+1}\,dx$		
3. $\int \frac{x}{\sqrt{x^2+1}}\,dx$		
4. $\int \sec 2x \tan 2x\,dx$		
5. $\int \tan^2 x \sec^2 x\,dx$		
6. $\int \frac{\cos x}{\sin^2 x}\,dx$		

In Exercises 7–26, evaluate the indefinite integral and check the result by differentiation.

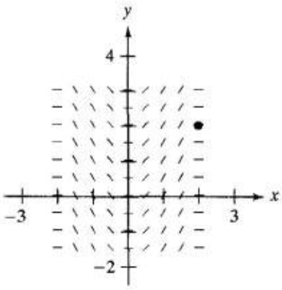
7. $\int (1+2x)^4(2)\,dx$
8. $\int (x^2-1)^3(2x)\,dx$
9. $\int \sqrt{9-x^2}\,(-2x)\,dx$
10. $\int (1-2x^2)^3(-4x)\,dx$
11. $\int x^2(x^3-1)^4\,dx$
12. $\int x(4x^2+3)^3\,dx$
13. $\int 5x\sqrt[3]{1-x^2}\,dx$
14. $\int u^3\sqrt{u^4+2}\,du$
15. $\int \frac{x^2}{(1+x^3)^2}\,dx$
16. $\int \frac{x^2}{(16-x^3)^2}\,dx$
17. $\int \left(1+\frac{1}{t}\right)^3\left(\frac{1}{t^2}\right)dt$
18. $\int \left[x^2+\frac{1}{(3x)^2}\right]dx$
19. $\int \frac{1}{\sqrt{2x}}\,dx$
20. $\int \frac{1}{2\sqrt{x}}\,dx$
21. $\int \frac{x^2+3x+7}{\sqrt{x}}\,dx$
22. $\int \frac{t+2t^2}{\sqrt{t}}\,dt$
23. $\int t^2\left(t-\frac{2}{t}\right)dt$
24. $\int \left(\frac{t^3}{3}+\frac{1}{4t^2}\right)dt$
25. $\int (9-y)\sqrt{y}\,dy$
26. $\int 2\pi y(8-y^{3/2})\,dy$

In Exercises 27–30, solve the differential equation.

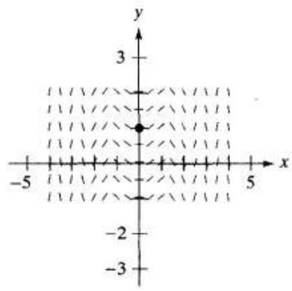
27. $\frac{dy}{dx} = 4x + \frac{4x}{\sqrt{16-x^2}}$
28. $\frac{dy}{dx} = \frac{10x^2}{\sqrt{1+x^3}}$
29. $\frac{dy}{dx} = \frac{x+1}{(x^2+2x-3)^2}$
30. $\frac{dy}{dx} = \frac{x-4}{\sqrt{x^2-8x+1}}$

Direction Fields **In Exercises 31 and 32, a differential equation, a point, and a direction field are given. A *direction field* consists of line segments with slopes given by the differential equation. These line segments give a visual perspective of the directions of the solutions of the differential equation. (a) Sketch two approximate solutions of the differential equation on the direction field, one of which passes through the indicated point. (b) Use integration to find the particular solution of the differential equation and use a graphing utility to graph the solution. Compare the result with the sketches in part (a).**

31. $\frac{dy}{dx} = x\sqrt{4-x^2}$, $(2, 2)$



32. $\frac{dy}{dx} = x\cos x^2$, $(0, 1)$



In Exercises 33–44, evaluate the indefinite integral.

33. $\int \sin 2x\,dx$
34. $\int x \sin x^2\,dx$
35. $\int \frac{1}{\theta^2}\cos\frac{1}{\theta}\,d\theta$
36. $\int \cos 6x\,dx$
37. $\int \sin 2x \cos 2x\,dx$
38. $\int \sec(1-x)\tan(1-x)\,dx$
39. $\int \tan^4 x \sec^2 x\,dx$
40. $\int \sqrt{\cot x}\,\csc^2 x\,dx$
41. $\int \frac{\csc^2 x}{\cot^3 x}\,dx$
42. $\int \frac{\sin x}{\cos^2 x}\,dx$
43. $\int \cot^2 x\,dx$
44. $\int \csc^2\left(\frac{x}{2}\right)dx$

In Exercises 45 and 46, find an equation for the function f that has the indicated derivative and whose graph passes through the given point.

Derivative	*Point*
45. $f'(x) = \cos\frac{x}{2}$	$(0, 3)$
46. $f'(x) = \pi\sec\pi x\tan\pi x$	$\left(\frac{1}{3}, 1\right)$

In Exercises 47–54, evaluate the indefinite integral by the method shown in Example 5.

47. $\int x\sqrt{x+2}\,dx$

48. $\int x\sqrt{2x+1}\,dx$

49. $\int x^2\sqrt{1-x}\,dx$

50. $\int (x+1)\sqrt{2-x}\,dx$

51. $\int \frac{x^2-1}{\sqrt{2x-1}}\,dx$

52. $\int \frac{2x-1}{\sqrt{x+3}}\,dx$

53. $\int \frac{-x}{(x+1)-\sqrt{x+1}}\,dx$

54. $\int t\sqrt[3]{t-4}\,dt$

In Exercises 55–64, evaluate the definite integral. Use a graphing utility to verify your result.

55. $\int_{-1}^{1} x(x^2+1)^3\,dx$

56. $\int_{0}^{1} x\sqrt{1-x^2}\,dx$

57. $\int_{0}^{4} \frac{1}{\sqrt{2x+1}}\,dx$

58. $\int_{0}^{2} \frac{x}{\sqrt{1+2x^2}}\,dx$

59. $\int_{1}^{9} \frac{1}{\sqrt{x}(1+\sqrt{x})^2}\,dx$

60. $\int_{0}^{2} x\sqrt[3]{4+x^2}\,dx$

61. $\int_{1}^{2} (x-1)\sqrt{2-x}\,dx$

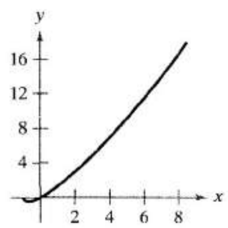
62. $\int_{0}^{4} \frac{x}{\sqrt{2x+1}}\,dx$

63. $\int_{0}^{\pi/2} \cos\left(\frac{2x}{3}\right)dx$

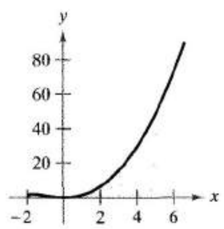
64. $\int_{\pi/3}^{\pi/2} (x+\cos x)\,dx$

In Exercises 65–70, find the area of the region. Use a graphing utility to verify your result.

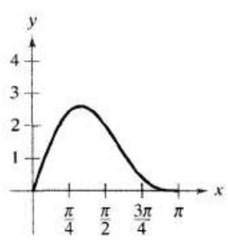
65. $\int_{0}^{7} x\sqrt[3]{x+1}\,dx$



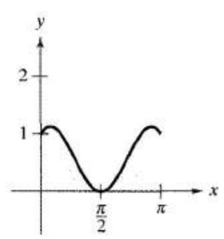
66. $\int_{-2}^{6} x^2\sqrt[3]{x+2}\,dx$



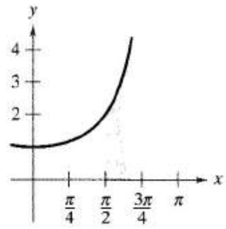
67. $y = 2\sin x + \sin 2x$



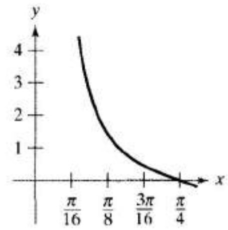
68. $y = \sin x + \cos 2x$



69. $\int_{\pi/2}^{2\pi/3} \sec^2\left(\frac{x}{2}\right)dx$



70. $\int_{\pi/12}^{\pi/4} \csc 2x \cot 2x\,dx$



In Exercises 71–76, use a graphing utility to evaluate the integral. Graph the region whose area is given by the definite integral.

71. $\int_{0}^{4} \frac{x}{\sqrt{2x+1}}\,dx$

72. $\int_{0}^{2} x^3\sqrt{x+2}\,dx$

73. $\int_{3}^{7} x\sqrt{x-3}\,dx$

74. $\int_{1}^{5} x^2\sqrt{x-1}\,dx$

75. $\int_{0}^{3} \left(\theta + \cos\frac{\theta}{6}\right)d\theta$

76. $\int_{0}^{\pi/2} \sin 2x\,dx$

Writing **In Exercises 77 and 78, perform the integration in two ways. Explain any difference in the forms of the answers.**

77. $\int (2x-1)^2\,dx$

78. $\int \sin x \cos x\,dx$

79. Use $\int_0^2 x^2\,dx = \frac{8}{3}$ to evaluate the definite integrals without using the Fundamental Theorem of Calculus.

(a) $\int_{-2}^{0} x^2\,dx$

(b) $\int_{-2}^{2} x^2\,dx$

(c) $\int_{0}^{2} -x^2\,dx$

(d) $\int_{-2}^{0} 3x^2\,dx$

80. Use the symmetry of the graphs of the sine and cosine functions as an aid in evaluating each of the integrals.

(a) $\int_{-\pi/4}^{\pi/4} \sin x\,dx$

(b) $\int_{-\pi/4}^{\pi/4} \cos x\,dx$

(c) $\int_{-\pi/2}^{\pi/2} \cos x\,dx$

(d) $\int_{-\pi/2}^{\pi/2} \sin x \cos x\,dx$

In Exercises 81 and 82, write the integral as the sum of the integral of an odd function and the integral of an even function. Use this simplification to evaluate the integral.

81. $\int_{-4}^{4} (x^3 + 6x^2 - 2x - 3)\,dx$

82. $\int_{-\pi}^{\pi} (\sin 3x + \cos 3x)\,dx$

83. ***Depreciation*** The rate of depreciation dV/dt of a machine is inversely proportional to the square of $t + 1$, where V is the value of the machine t years after it was purchased. If the initial value of the machine was \$500,000, and its value decreased \$100,000 in the first year, estimate its value after 4 years.

84. ***Cash Flow*** The rate of disbursement dQ/dt of a 2 million dollar federal grant is proportional to the square of $100 - t$. Time t is measured in days $(0 \le t \le 100)$, and Q is the amount that remains to be disbursed. Find the amount that remains to be disbursed after 50 days. Assume that all the money will be disbursed in 100 days.

85. ***Marginal Cost*** The marginal cost for a certain commodity has been determined to be

$$\frac{dC}{dx} = \frac{12}{\sqrt[3]{12x + 1}}.$$

(a) Find the cost function if $C = 100$ when $x = 13$.

(b) Use a graphing utility to graph the marginal cost function and the cost function in the same viewing rectangle.

86. ***Gasoline Reserves*** The minimum stockpile level of gasoline in the United States can be approximated by the model

$$Q = 217 + 13 \cos \frac{\pi(t - 3)}{6}$$

where Q is measured in millions of barrels of gasoline and t is the time in months, with $t = 1$ corresponding to January. Find the average minimum level given by this model during the following periods.

(a) The first quarter $(0 \le t \le 3)$

(b) The second quarter $(3 \le t \le 6)$

(c) The entire year $(0 \le t \le 12)$

87. ***Sales*** The sales of a seasonal product are given by the model

$$S = 74.50 + 43.75 \sin \frac{\pi t}{6}$$

where S is measured in thousands of units and t is the time in months, with $t = 1$ corresponding to January. Find the average sales for the following periods.

(a) The first quarter $(0 \le t \le 3)$

(b) The second quarter $(3 \le t \le 6)$

(c) The entire year $(0 \le t \le 12)$

88. ***Water Supply*** A model for the flow rate of water at a pumping station on a given day is

$$R(t) = 53 + 7 \sin\left(\frac{\pi t}{6} + 3.6\right) + 9 \cos\left(\frac{\pi t}{12} + 8.9\right)$$

where $0 \le t \le 24$. R is the flow rate in thousands of gallons per hour, and t is the time in hours.

(a) Use a graphing utility to graph the rate function and approximate the maximum flow rate at the pumping station.

(b) Approximate the total volume of water pumped in 1 day.

89. ***Electricity*** The oscillating current in an electrical circuit is

$$I = 2 \sin(60\pi t) + \cos(120\pi t)$$

where I is measured in amperes and t is measured in seconds. Find the average current for each of the following time intervals.

(a) $0 \le t \le \frac{1}{60}$ (b) $0 \le t \le \frac{1}{240}$ (c) $0 \le t \le \frac{1}{30}$

90. ***Graphical Analysis*** Consider the functions f and g, where

$$f(x) = 6 \sin x \cos^2 x \quad \text{and} \quad g(t) = \int_0^t f(x)\, dx.$$

(a) Use a graphing utility to graph f and g in the same viewing rectangle.

(b) Explain why g is nonnegative.

(c) Identify the points on the graph of g that correspond to the extrema of f.

(d) Does each of the zeros of f correspond to an extrema of g? Explain.

(e) Consider the function $h(t) = \int_{\pi/2}^{t} f(x)\, dx$. Use a graphing utility to graph h. What is the relationship between g and h? Verify your conjecture.

True or False? **In Exercises 91–96, determine whether the statement is true or false. If it is false, explain why or give an example that shows it is false.**

91. $\displaystyle\int (2x + 1)^2\, dx = \frac{1}{3}(2x + 1)^3 + C$

92. $\displaystyle\int x(x^2 + 1)\, dx = \frac{1}{2}x^2\left(\frac{1}{3}x^3 + x\right) + C$

93. $\displaystyle\int_{-10}^{10} (ax^3 + bx^2 + cx + d)\, dx = 2\int_0^{10} (bx^2 + d)\, dx$

94. $\displaystyle\int_a^b \sin x\, dx = \int_a^{b+2\pi} \sin x\, dx$

95. $\displaystyle 4\int \sin x \cos x\, dx = -\cos 2x + C$

96. $\displaystyle\int \sin^2 2x \cos 2x\, dx = \frac{1}{3}\sin^3 2x + C$

97. Show that if f is continuous on the entire real line, then

$$\int_a^b f(x + h)\, dx = \int_{a+h}^{b+h} f(x)\, dx.$$

98. Let f be continuous on the interval $[0, b]$. Show that

$$\int_0^b \frac{f(x)}{f(x) + f(b - x)}\, dx = \frac{b}{2}.$$

Use this result to evaluate

$$\int_0^1 \frac{\sin x}{\sin(1 - x) + \sin x}\, dx.$$