EXERCISES FOR SECTION 5.2

In Exercises 1–18, find the indefinite integral.

1. $\int \frac{1}{x+1}\,dx$
2. $\int \frac{1}{x-5}\,dx$
3. $\int \frac{1}{3-2x}\,dx$
4. $\int \frac{1}{6x+1}\,dx$
5. $\int \frac{x}{x^2+1}\,dx$
6. $\int \frac{x^2}{3-x^3}\,dx$
7. $\int \frac{x^2-4}{x}\,dx$
8. $\int \frac{x}{\sqrt{9-x^2}}\,dx$
9. $\int \frac{x^2+2x+3}{x^3+3x^2+9x}\,dx$
10. $\int \frac{x+3}{x^2+6x+7}\,dx$
11. $\int \frac{(\ln x)^2}{x}\,dx$
12. $\int \frac{1}{x\ln(x^2)}\,dx$
13. $\int \frac{1}{\sqrt{x+1}}\,dx$
14. $\int \frac{1}{x^{2/3}(1+x^{1/3})}\,dx$
15. $\int \frac{\sqrt{x}}{\sqrt{x}-3}\,dx$
16. $\int \frac{1}{1+\sqrt{2x}}\,dx$
17. $\int \frac{2x}{(x-1)^2}\,dx$
18. $\int \frac{x(x-2)}{(x-1)^3}\,dx$

In Exercises 19–26, find the indefinite integral of the trigonometric function.

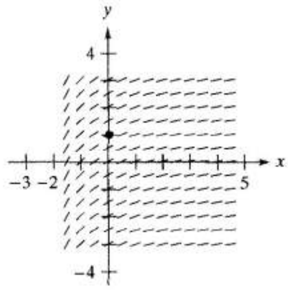
19. $\int \frac{\cos\theta}{\sin\theta}\,d\theta$
20. $\int \tan 5\theta\,d\theta$
21. $\int \csc 2x\,dx$
22. $\int \sec\frac{x}{2}\,dx$
23. $\int \frac{\cos t}{1+\sin t}\,dt$
24. $\int \frac{\sin x}{1+\cos x}\,dx$
25. $\int \frac{\sec x\tan x}{\sec x-1}\,dx$
26. $\int (\sec t+\tan t)\,dt$

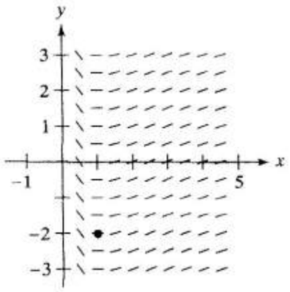
In Exercises 27–30, solve the differential equation. Use a graphing utility to graph three solutions, one of which passes through the indicated point.

27. $\frac{dy}{dx} = \frac{3}{2-x}$, $(1, 0)$
28. $\frac{dy}{dx} = \frac{2x}{x^2-9}$, $(0, 4)$
29. $\frac{ds}{d\theta} = \tan 2\theta$, $(0, 2)$
30. $\frac{dr}{dt} = \frac{\sec^2 t}{\tan t+1}$, $(\pi, 4)$

Direction Fields **In Exercises 31 and 32, a differential equation, a point, and a direction field are given. (a) Sketch two approximate solutions of the differential equation on the direction field, one of which passes through the indicated point. (b) Use integration to find the particular solution of the differential equation and use a graphing utility to graph the solution. Compare the result with the sketches in part (a).**

31. $\frac{dy}{dx} = \frac{1}{x+2}$, $(0, 1)$
32. $\frac{dy}{dx} = \frac{\ln x}{x}$, $(1, -2)$





In Exercises 33–40, evaluate the definite integral. Use a graphing utility to verify your result.

33. $\int_0^4 \frac{5}{3x+1}\,dx$
34. $\int_{-1}^1 \frac{1}{x+2}\,dx$
35. $\int_1^e \frac{(1+\ln x)^2}{x}\,dx$
36. $\int_e^{e^2} \frac{1}{x\ln x}\,dx$
37. $\int_0^2 \frac{x^2-2}{x+1}\,dx$
38. $\int_0^1 \frac{x-1}{x+1}\,dx$
39. $\int_1^2 \frac{1-\cos\theta}{\theta-\sin\theta}\,d\theta$
40. $\int_{0.1}^{0.2} (\csc 2\theta-\cot 2\theta)^2\,d\theta$

In Exercises 41–46, use a symbolic integration utility to evaluate the integral. Graph the integrand.

41. $\int \frac{1}{1+\sqrt{x}}\,dx$
42. $\int \frac{1-\sqrt{x}}{1+\sqrt{x}}\,dx$
43. $\int \cos(1-x)\,dx$
44. $\int \frac{\tan^2 2x}{\sec 2x}\,dx$
45. $\int_{\pi/4}^{\pi/2} (\csc x-\sin x)\,dx$
46. $\int_{-\pi/4}^{\pi/4} \frac{\sin^2 x-\cos^2 x}{\cos x}\,dx$

In Exercises 47–50, show that the two formulas are equivalent.

47. $\int \tan x\,dx = -\ln|\cos x| + C$

$\int \tan x\,dx = \ln|\sec x| + C$

48. $\int \cot x\,dx = \ln|\sin x| + C$

$\int \cot x\,dx = -\ln|\csc x| + C$

49. $\int \sec x\,dx = \ln|\sec x + \tan x| + C$

$\int \sec x\,dx = -\ln|\sec x - \tan x| + C$

50. $\int \csc x\,dx = -\ln|\csc x + \cot x| + C$

$\int \csc x\,dx = \ln|\csc x - \cot x| + C$

In Exercises 51–54, find $F'(x)$.

51. $F(x) = \int_1^x \frac{1}{t}\,dt$ **52.** $F(x) = \int_0^x \tan t\,dt$

53. $F(x) = \int_x^{3x} \frac{1}{t}\,dt$ **54.** $F(x) = \int_1^{x^2} \frac{1}{t}\,dt$

Approximation **In Exercises 55 and 56, determine which value best approximates the area of the region between the x-axis and the function over the given interval. (Make your selection on the basis of a sketch of the region and *not* by performing any calculations.)**

55. $f(x) = \sec x$, $[0, 1]$

(a) 6 (b) -6 (c) $\frac{1}{2}$ (d) 1.25 (e) 3

56. $f(x) = \frac{2x}{x^2 + 1}$, $[0, 4]$

(a) 3 (b) 7 (c) -2 (d) 5 (e) 1

Area **In Exercises 57–60, find the area of the region bounded by the graphs of the equations. Use a graphing utility to graph the region and verify your result.**

57. $y = \frac{x^2 + 4}{x}, x = 1, x = 4, y = 0$

58. $y = \frac{x + 5}{x}, x = 1, x = 5, y = 0$

59. $y = 2\sec\frac{\pi x}{6}, x = 0, x = 2, y = 0$

60. $y = 2x - \tan(0.3x), x = 1, x = 4, y = 0$

61. ***Population Growth*** A population of bacteria is changing at a rate of

$$\frac{dP}{dt} = \frac{3000}{1 + 0.25t}$$

where t is the time in days. The initial population (when $t = 0$) is 1000. Write an equation that gives the population at any time t, and find the population when $t = 3$ days.

62. ***Heat Transfer*** Find the time required for an object to cool from 300°F to 250°F by evaluating

$$t = \frac{10}{\ln 2}\int_{250}^{300} \frac{1}{T - 100}\,dT$$

where t is time in minutes.

63. ***Average Price*** The demand equation for a product is

$$p = \frac{90{,}000}{400 + 3x}.$$

Find the *average* price p on the interval $40 \le x \le 50$.

64. ***Sales*** The rate of change in sales S is inversely proportional to time t ($t > 1$) measured in weeks. Find S as a function of t if sales after 2 and 4 weeks are 200 units and 300 units.

65. ***Orthogonal Trajectory***

(a) Use a graphing utility to graph the equation $2x^2 - y^2 = 8$.

(b) Evaluate the integral to find y^2 in terms of x.

$y^2 = e^{-\int(1/x)\,dx}$

For a particular value of the constant of integration, graph the result on the same screen used in part (a).

(c) Verify that the tangents to the graphs of parts (a) and (b) are perpendicular at the points of intersection.

66. Graph the function

$$f_k(x) = \frac{x^k - 1}{k}$$

for $k = 1, 0.5$, and 0.1 on $[0, 10]$. Find $\lim_{k \to 0^+} f_k(x)$.

True or False? **In Exercises 67–70, determine whether the statement is true or false. If it is false, explain why or give an example that shows it is false.**

67. $(\ln x)^{1/2} = \frac{1}{2}(\ln x)$

68. $\int \ln x\,dx = (1/x) + C$

69. $\int \frac{1}{x}\,dx = \ln|cx|, \quad c \ne 0$

70. $\int_{-1}^{2} \frac{1}{x}\,dx = \Big[\ln|x|\Big]_{-1}^{2} = \ln 2 - \ln 1 = \ln 2$