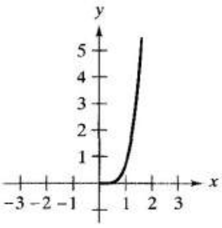
EXERCISES FOR SECTION 5.3

In Exercises 1–8, show that f and g are inverse functions (a) algebraically and (b) graphically.

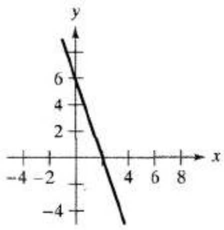
1. $f(x) = 5x + 1$, $g(x) = (x - 1)/5$
2. $f(x) = 3 - 4x$, $g(x) = (3 - x)/4$
3. $f(x) = x^3$, $g(x) = \sqrt[3]{x}$
4. $f(x) = 1 - x^3$, $g(x) = \sqrt[3]{1 - x}$
5. $f(x) = \sqrt{x - 4}$, $g(x) = x^2 + 4, \quad x \geq 0$
6. $f(x) = 9 - x^2, \quad x \geq 0$, $g(x) = \sqrt{9 - x}$
7. $f(x) = 1/x$, $g(x) = 1/x$
8. $f(x) = \dfrac{1}{1 + x}, \quad x \geq 0$, $g(x) = \dfrac{1 - x}{x}, \quad 0 < x \leq 1$

In Exercises 9–12, match the graph of the function with the graph of its inverse. [The graphs of the inverse functions are labeled (a), (b), (c), and (d).]

(a)



(b)



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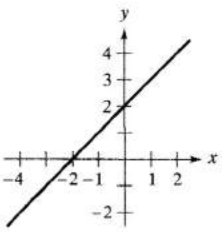
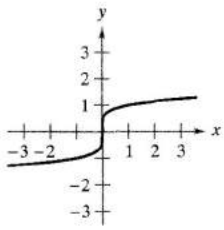


(d)



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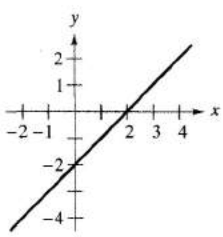

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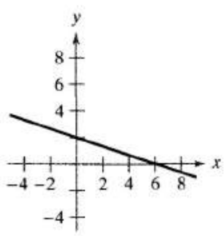

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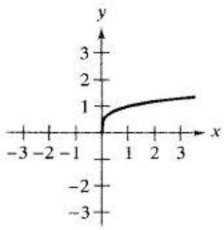

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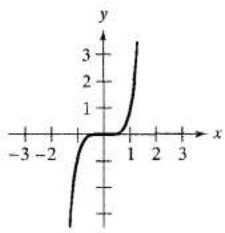


In Exercises 13–20, find the inverse of f. Graph (by hand) f and f^{-1}. Decribe the relationship between the graphs.

13. $f(x) = 2x - 3$
14. $f(x) = 3x$
15. $f(x) = x^5$
16. $f(x) = x^3 + 1$
17. $f(x) = \sqrt{x}$
18. $f(x) = x^2, \quad x \geq 0$
19. $f(x) = \sqrt{4 - x^2}, \quad x \geq 0$
20. $f(x) = \sqrt{x^2 - 4}, \quad x \geq 2$

In Exercises 21–26, find the inverse of f. Use a graphing utility to graph f and f^{-1} in the same viewing rectangle. Describe the relationship between the graphs.

21. $f(x) = \sqrt[3]{x - 1}$
22. $f(x) = 3\sqrt[5]{2x - 1}$
23. $f(x) = x^{2/3}, \quad x \geq 0$
24. $f(x) = x^{3/5}$
25. $f(x) = \dfrac{x}{\sqrt{x^2 + 7}}$
26. $f(x) = \dfrac{x + 2}{x}$

In Exercises 27 and 28, find the inverse function of f over the indicated interval. Use a graphing utility to graph f and f^{-1} in the same viewing rectangle. Describe the relationship between the graphs.

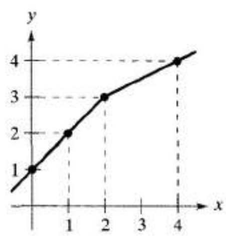
	Function	*Interval*
27.	$f(x) = \dfrac{x}{x^2 - 4}$	$-2 < x < 2$
28.	$f(x) = 2 - \dfrac{3}{x^2}$	$0 < x < 10$

Graphical Reasoning **In Exercises 29–32, (a) use a graphing utility to graph the function, (b) use the drawing feature of a graphing utility to draw the inverse of the function, and (c) determine whether the graph of the inverse relation is an inverse function. Explain your reasoning.**

29. $f(x) = x^3 + x + 4$
30. $h(x) = x\sqrt{4 - x^2}$
31. $g(x) = \dfrac{3x^2}{x^2 + 1}$
32. $f(x) = \dfrac{4x}{\sqrt{x^2 + 15}}$

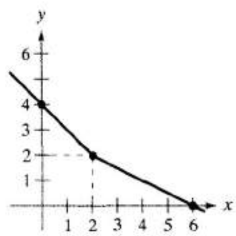
In Exercises 33 and 34, use the graph of the function f to complete the table and sketch the graph of f^{-1}.

33.



x	1	2	3	4
$f^{-1}(x)$				

34.



x	0	2	4
$f^{-1}(x)$			

35. ***Cost*** Suppose you need 50 pounds of two commodities costing \$1.25 and \$1.60 per pound.

(a) Verify that the total cost is

$$y = 1.25x + 1.60(50 - x)$$

where x is the number of pounds of the less expensive commodity.

(b) Find the inverse of the cost function. What does each variable represent in the inverse function?

(c) Use the context of the problem to determine the domain of the inverse function.

(d) Determine the number of pounds of the less expensive commodity purchased if the total cost is \$73.

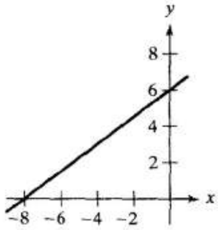
36. ***Think About It*** The function

$$f(x) = k(2 - x - x^3)$$

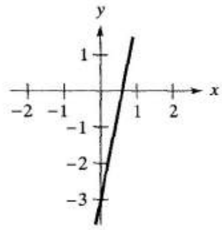
is one-to-one and $f^{-1}(3) = -2$. Find k.

In Exercises 37–40, use the horizontal line test to determine whether the function is one-to-one on its entire domain and therefore has an inverse.

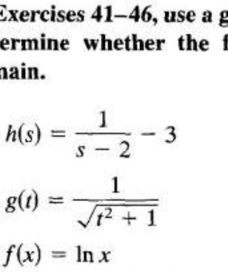
37. $f(x) = \frac{3}{4}x + 6$



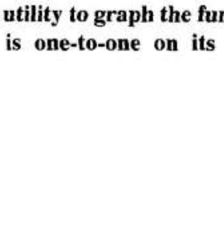
38. $f(x) = 5x - 3$



39. $f(\theta) = \sin\theta$



40. $f(x) = \dfrac{x^2}{x^2 + 4}$



In Exercises 41–46, use a graphing utility to graph the function. Determine whether the function is one-to-one on its entire domain.

41. $h(s) = \dfrac{1}{s - 2} - 3$

42. $g(t) = \dfrac{1}{\sqrt{t^2 + 1}}$

43. $f(x) = \ln x$

44. $f(x) = 3x\sqrt{x + 1}$

45. $g(x) = (x + 5)^3$

46. $h(x) = |x + 4| - |x - 4|$

In Exercises 47–52, use the derivative to determine whether the function is strictly monotonic on its entire domain and therefore has an inverse.

47. $f(x) = (x + a)^3 + b$

48. $f(x) = \cos\dfrac{3x}{2}$

49. $f(x) = \dfrac{x^4}{4} - 2x^2$

50. $f(x) = x^3 - 6x^2 + 12x$

51. $f(x) = 2 - x - x^3$

52. $f(x) = \ln(x - 3)$

In Exercises 53–58, show that f is strictly monotonic on the indicated interval and therefore has an inverse on that interval.

	Function	*Interval*
53.	$f(x) = (x - 4)^2$	$[4, \infty)$
54.	$f(x) = \lvert x + 2\rvert$	$[-2, \infty)$
55.	$f(x) = \dfrac{4}{x^2}$	$(0, \infty)$
56.	$f(x) = \tan x$	$\left(-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right)$
57.	$f(x) = \cos x$	$[0, \pi]$
58.	$f(x) = \sec x$	$\left[0, \dfrac{\pi}{2}\right)$

Think About It **In Exercises 59 and 60, the derivative of the function has the same sign for all x in its domain, but the function is not one-to-one. Explain.**

59. $f(x) = \tan x$

60. $f(x) = \dfrac{x}{x^2 - 4}$

In Exercises 61–64, determine whether the function is one-to-one. If it is, find its inverse.

61. $f(x) = \sqrt{x - 2}$

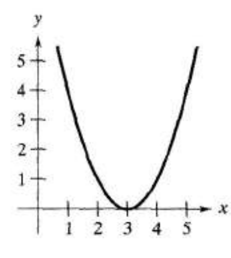
62. $f(x) = -3$

63. $f(x) = |x - 2|, \quad x \le 2$

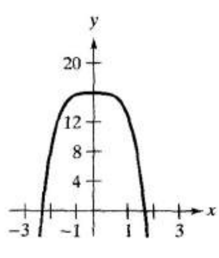
64. $f(x) = ax + b, \quad a \ne 0$

In Exercises 65–68, delete part of the graph of the function so that the part that remains is one-to-one. Find the inverse of the remaining part and give the domain of the inverse. (*Note:* There is more than one correct answer.)

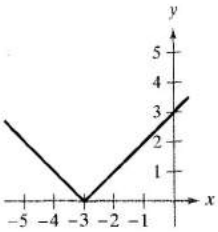
65. $f(x) = (x - 3)^2$



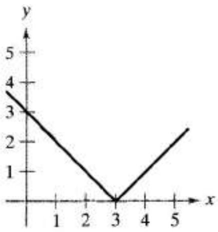
66. $f(x) = 16 - x^4$



67. $f(x) = |x + 3|$



68. $f(x) = |x - 3|$



Think About It **In Exercises 69–72, decide whether the function has an inverse. If so, what is the inverse?**

69. $g(t)$ is the volume of water that has passed through a water line t minutes after a control valve is opened.

70. $h(t)$ is the height of the tide t hours after midnight, where $0 \le t < 24$.

71. $C(t)$ is the cost of a long distance call lasting t minutes.

72. $A(r)$ is the area of a circle of radius r.

In Exercises 73–78, find $(f^{-1})'(a)$ for the function f and real number a.

Function	*Real Number*
73. $f(x) = x^3 + 2x - 1$	$a = 2$
74. $f(x) = 2x^5 + x^3 + 1$	$a = -2$
75. $f(x) = \sin x, \quad -\frac{\pi}{2} \le x \le \frac{\pi}{2}$	$a = \frac{1}{2}$
76. $f(x) = \cos 2x, \quad 0 \le x \le \frac{\pi}{2}$	$a = 1$
77. $f(x) = x^3 - \frac{4}{x}$	$a = 6$
78. $f(x) = \sqrt{x - 4}$	$a = 2$

In Exercises 79–82, (a) find the domains of f and f^{-1}, (b) find the ranges of f and f^{-1}, (c) graph f and f^{-1}, and (d) show that the slopes of the graphs of f and f^{-1} are reciprocals at the indicated points.

Functions	*Point*
79. $f(x) = x^3$	$\left(\frac{1}{2}, \frac{1}{8}\right)$
$f^{-1}(x) = \sqrt[3]{x}$	$\left(\frac{1}{8}, \frac{1}{2}\right)$
80. $f(x) = 3 - 4x$	$(1, -1)$
$f^{-1}(x) = \frac{3 - x}{4}$	$(-1, 1)$
81. $f(x) = \sqrt{x - 4}$	$(5, 1)$
$f^{-1}(x) = x^2 + 4$	$(1, 5)$
82. $f(x) = \frac{1}{1 + x^2}, \quad x \ge 0$	$\left(1, \frac{1}{2}\right)$
$f^{-1}(x) = \sqrt{\frac{1 - x}{x}}$	$\left(\frac{1}{2}, 1\right)$

In Exercises 83 and 84, find dy/dx at the indicated point for the equation.

83. $x = y^3 - 7y^2 + 2$
$(-4, 1)$

84. $x = 2 \ln(y^2 - 3)$
$(0, 4)$

In Exercises 85–88, use the functions

$$f(x) = \tfrac{1}{8}x - 3 \quad \text{and} \quad g(x) = x^3$$

to find the indicated value.

85. $(f^{-1} \circ g^{-1})(1)$

86. $(g^{-1} \circ f^{-1})(-3)$

87. $(f^{-1} \circ f^{-1})(6)$

88. $(g^{-1} \circ g^{-1})(-4)$

In Exercises 89–92, use the functions

$$f(x) = x + 4 \quad \text{and} \quad g(x) = 2x - 5$$

to find the indicated function.

89. $g^{-1} \circ f^{-1}$

90. $f^{-1} \circ g^{-1}$

91. $(f \circ g)^{-1}$

92. $(g \circ f)^{-1}$

93. Prove that if f and g are one-to-one functions, then $(f \circ g)^{-1}(x) = (g^{-1} \circ f^{-1})(x)$.

94. Prove that if f has an inverse, then $(f^{-1})^{-1} = f$.

95. Prove that if a function has an inverse, then the inverse is unique.

96. Prove that a function has an inverse if and only if it is one-to-one.

True or False? **In Exercises 97–100, determine whether the statement is true or false. If it is false, explain why or give an example that shows it is false.**

97. If f is an even function, then f^{-1} exists.

98. If the inverse of f exists, then the y-intercept of f is an x-intercept of f^{-1}.

99. If $f(x) = x^n$ where n is odd, then f^{-1} exists.

100. There exists no function f such that $f = f^{-1}$.

101. Is the converse of the second part of Theorem 5.7 true? That is, if a function is one-to-one (and hence has an inverse), then must the function be strictly monotonic? If so, prove it. If not, give a counterexample.

102. Let f be twice-differentiable and one-to-one on an open interval I. Show that its inverse g satisfies

$$g''(x) = -\frac{f''(g(x))}{[f'(g(x))]^3}.$$

If f is increasing and concave downward, what is the concavity of $f^{-1} = g$?

103. If $f(x) = \int_2^x \frac{dt}{\sqrt{1 + t^4}}$, find $(f^{-1})'(0)$.