EXERCISES FOR SECTION 5.8

Numerical and Graphical Analysis **In Exercises 1 and 2, (a) use a graphing utility to complete the table, (b) plot the points in the table and graph the function by hand, (c) use a graphing utility to graph the function and compare the result with your hand-drawn graph in part (b), and (d) determine any intercepts and symmetry of the graph.**

x	-1	-0.8	-0.6	-0.4	-0.2
y					

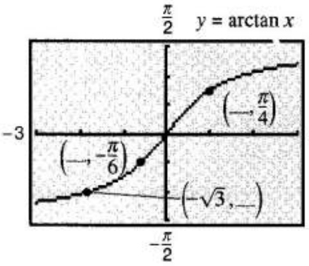
x	0	0.2	0.4	0.6	0.8	1
y						

1. $y = \arcsin x$

2. $y = \arccos x$

3. ***True or False?*** Decide whether the following statement is true or false, and explain: Because $\cos(-\pi/3) = \frac{1}{2}$, it follows that $\arccos \frac{1}{2} = -\pi/3$.

4. Determine the missing coordinates of the points on the graph of the function.



In Exercises 5–12, evaluate the expression without using a calculator.

5. $\arcsin \frac{1}{2}$

6. $\arcsin 0$

7. $\arccos \frac{1}{2}$

8. $\arccos 0$

9. $\arctan \dfrac{\sqrt{3}}{3}$

10. $\operatorname{arccot}(-1)$

11. $\operatorname{arccsc}(-\sqrt{2})$

12. $\arccos\left(-\dfrac{\sqrt{3}}{2}\right)$

In Exercises 13–16, use a calculator to approximate the inverse trigonometric function. Round your answer to two decimal places.

13. $\arccos(-0.8)$

14. $\arcsin(-0.39)$

15. $\operatorname{arcsec} 1.269$

16. $\arctan(-3)$

17. ***Think About It*** Explain why $\tan \pi = 0$ does not imply that $\arctan 0 = \pi$.

18. Use a graphing utility to confirm that $f(x) = \sin x$ and $g(x) = \arcsin x$ are inverse functions. (Remember to restrict the domain of f properly.)

In Exercises 19–22, evaluate the expression without using a calculator. (*Hint:* See Example 3.)

19. (a) $\sin\left(\arctan \dfrac{3}{4}\right)$

(b) $\sec\left(\arcsin \dfrac{4}{5}\right)$

20. (a) $\tan\left(\arccos \dfrac{\sqrt{2}}{2}\right)$

(b) $\cos\left(\arcsin \dfrac{5}{13}\right)$

21. (a) $\cot\left[\arcsin\left(-\dfrac{1}{2}\right)\right]$

(b) $\csc\left[\arctan\left(-\dfrac{5}{12}\right)\right]$

22. (a) $\sec\left[\arctan\left(-\dfrac{3}{5}\right)\right]$

(b) $\tan\left[\arcsin\left(-\dfrac{5}{6}\right)\right]$

In Exercises 23–30, write the expression in algebraic form.

23. $\cos(\arcsin 2x)$

24. $\sec(\arctan 3x)$

25. $\sin(\operatorname{arcsec} x)$

26. $\cos(\operatorname{arccot} x)$

27. $\tan\left(\operatorname{arcsec} \dfrac{x}{3}\right)$

28. $\sec[\arcsin(x - 1)]$

29. $\csc\left(\arctan \dfrac{x}{\sqrt{2}}\right)$

30. $\cos\left(\arcsin \dfrac{x - h}{r}\right)$

In Exercises 31 and 32, use a graphing utility to graph f and g in the same viewing rectangle to verify that they are equal. Explain why they are equal. Identify any asymptotes of the graphs.

31. $f(x) = \sin(\arctan 2x), \quad g(x) = \dfrac{2x}{\sqrt{1 + 4x^2}}$

32. $f(x) = \tan\left(\arccos \dfrac{x}{2}\right), \quad g(x) = \dfrac{\sqrt{4 - x^2}}{x}$

In Exercises 33 and 34, verify each identity.

33. (a) $\operatorname{arccsc} x = \arcsin \dfrac{1}{x}, \quad |x| \geq 1$

(b) $\arctan x + \arctan \dfrac{1}{x} = \dfrac{\pi}{2}, \quad x > 0$

34. (a) $\arcsin(-x) = -\arcsin x, \quad |x| \leq 1$

(b) $\arccos(-x) = \pi - \arccos x, \quad |x| \leq 1$

In Exercises 35–38, sketch the graph of the function. Use a graphing utility to verify your graph.

35. $f(x) = \arcsin(x - 1)$

36. $f(x) = \arctan x + \dfrac{\pi}{2}$

37. $f(x) = \operatorname{arcsec} 2x$

38. $f(x) = \arccos \dfrac{x}{4}$

In Exercises 39–42, solve the equation for x.

39. $\arcsin(3x - \pi) = \frac{1}{2}$

40. $\arctan 2x = -1$

41. $\arcsin\sqrt{2x} = \arccos\sqrt{x}$

42. $\arccos x = \operatorname{arcsec} x$

In Exercises 43–56, find the derivative of the function.

43. $f(x) = 2\arcsin(x - 1)$

44. $f(t) = \arcsin t^2$

45. $g(x) = 3\arccos\frac{x}{2}$

46. $f(x) = \operatorname{arcsec} 2x$

47. $f(x) = \arctan\frac{x}{a}$

48. $f(x) = \arctan\sqrt{x}$

49. $g(x) = \frac{\arcsin 3x}{x}$

50. $h(x) = x\arctan x$

51. $h(t) = \sin(\arccos t)$

52. $f(x) = \arcsin x + \arccos x$

53. $y = \frac{1}{2}\left(\frac{1}{2}\ln\frac{x+1}{x-1} + \arctan x\right)$

54. $y = \frac{1}{2}\left(x\sqrt{1-x^2} + \arcsin x\right)$

55. $y = x\arcsin x + \sqrt{1-x^2}$

56. $y = x\arctan 2x - \frac{1}{4}\ln(1 + 4x^2)$

Linear and Quadratic Approximations **In Exercises 57 and 58, use a symbolic differentiation utility to find the linear approximation**

$$P_1(x) = f(a) + f'(a)(x - a)$$

and the quadratic approximation

$$P_2(x) = f(a) + f'(a)(x - a) + \tfrac{1}{2}f''(a)(x - a)^2$$

to the function f at $x = a$. Sketch the graph of the function and its linear and quadratic approximations.

57. $f(x) = \arcsin x$
$a = \frac{1}{2}$

58. $f(x) = \arctan x$
$a = 1$

In Exercises 59 and 60, find any relative extrema of the function.

59. $f(x) = \operatorname{arcsec} x - x$

60. $f(x) = \arcsin x - 2x$

61. ***Angular Rate of Change*** In a free-fall experiment, an object is dropped from a height of 256 feet. A camera on the ground 500 feet from the point of impact records the fall of the object.
    (a) Find the position function giving the height of the object at time t assuming the object is released at time $t = 0$. At what time will the object reach ground level?
    (b) Find the rate of change of the angle of elevation of the camera when $t = 1$ and $t = 2$.

62. ***Angular Rate of Change*** A television camera at ground level is filming the lift-off of a space shuttle at a point 750 meters from the launch pad. Let θ be the angle of elevation of the shuttle and let s be the distance between the camera and the shuttle. Write θ as a function of s for the period of time when the shuttle is moving vertically. Differentiate the result to find $d\theta/dt$ in terms of s and ds/dt.

63. Prove that

$$\arctan x + \arctan y = \arctan\frac{x+y}{1-xy}, \quad xy \neq 1.$$

Use this formula to show that

$$\arctan\frac{1}{2} + \arctan\frac{1}{3} = \frac{\pi}{4}.$$

64. Verify each of the following differentiation formulas.
    (a) $\frac{d}{dx}[\arcsin u] = \frac{u'}{\sqrt{1-u^2}}$
    (b) $\frac{d}{dx}[\arctan u] = \frac{u'}{1+u^2}$
    (c) $\frac{d}{dx}[\operatorname{arcsec} u] = \frac{u'}{|u|\sqrt{u^2-1}}$
    (d) $\frac{d}{dx}[\arccos u] = \frac{-u'}{\sqrt{1-u^2}}$
    (e) $\frac{d}{dx}[\operatorname{arccot} u] = \frac{-u'}{1+u^2}$
    (f) $\frac{d}{dx}[\operatorname{arccsc} u] = \frac{-u'}{|u|\sqrt{u^2-1}}$

65. ***Existence of an Inverse*** Determine the values of k such that the function

$$f(x) = kx + \sin x$$

has an inverse.

66. ***Think About It*** Use a graphing utility to graph

$$f(x) = \sin x \text{ and } g(x) = \arcsin(\sin x).$$

    (a) Why isn't the graph of g the line $y = x$?
    (b) Determine the extrema of g.

True or False? **In Exercises 67–70, determine whether the statement is true or false. If it is false, explain why or give an example that shows it is false.**

67. The slope of the graph of the inverse tangent is positive for all x.

68. The range of $y = \arcsin x$ is $[0, \pi]$.

69. $\frac{d}{dx}[\arctan(\tan x)] = 1$ for all x in the domain.

70. $\arcsin^2 x + \arccos^2 x = 1$