EXERCISES FOR SECTION 5.9

In Exercises 1–20, evaluate the integral.

1. $\displaystyle\int_0^{1/6} \frac{1}{\sqrt{1-9x^2}}\,dx$

2. $\displaystyle\int_0^{1} \frac{dx}{\sqrt{4-x^2}}$

3. $\displaystyle\int_0^{\sqrt{3}/2} \frac{1}{1+4x^2}\,dx$

4. $\displaystyle\int_{\sqrt{3}}^{3} \frac{1}{9+x^2}\,dx$

5. $\displaystyle\int \frac{1}{x\sqrt{4x^2-1}}\,dx$

6. $\displaystyle\int \frac{1}{4+(x-1)^2}\,dx$

7. $\displaystyle\int \frac{x^3}{x^2+1}\,dx$

8. $\displaystyle\int \frac{x^4-1}{x^2+1}\,dx$

9. $\displaystyle\int \frac{1}{\sqrt{1-(x+1)^2}}\,dx$

10. $\displaystyle\int \frac{t}{t^4+16}\,dt$

11. $\displaystyle\int \frac{t}{\sqrt{1-t^4}}\,dt$

12. $\displaystyle\int \frac{1}{x\sqrt{x^4-4}}\,dx$

13. $\displaystyle\int_0^{1/\sqrt{2}} \frac{\arcsin x}{\sqrt{1-x^2}}\,dx$

14. $\displaystyle\int_0^{1/\sqrt{2}} \frac{\arccos x}{\sqrt{1-x^2}}\,dx$

15. $\displaystyle\int_{-1/2}^{0} \frac{x}{\sqrt{1-x^2}}\,dx$

16. $\displaystyle\int_{-\sqrt{3}}^{0} \frac{x}{1+x^2}\,dx$

17. $\displaystyle\int \frac{e^{2x}}{4+e^{4x}}\,dx$

18. $\displaystyle\int_1^{2} \frac{1}{3+(x-2)^2}\,dx$

19. $\displaystyle\int_{\pi/2}^{\pi} \frac{\sin x}{1+\cos^2 x}\,dx$

20. $\displaystyle\int \frac{1}{\sqrt{x}(1+x)}\,dx$

In Exercises 21–32, evaluate the integral. (Complete the square, if necessary.)

21. $\displaystyle\int_0^{2} \frac{dx}{x^2-2x+2}$

22. $\displaystyle\int_{-3}^{-1} \frac{dx}{x^2+6x+13}$

23. $\displaystyle\int \frac{2x}{x^2+6x+13}\,dx$

24. $\displaystyle\int \frac{2x-5}{x^2+2x+2}\,dx$

25. $\displaystyle\int \frac{1}{\sqrt{-x^2-4x}}\,dx$

26. $\displaystyle\int \frac{1}{\sqrt{-x^2+2x}}\,dx$

27. $\displaystyle\int \frac{x+2}{\sqrt{-x^2-4x}}\,dx$

28. $\displaystyle\int \frac{x-1}{\sqrt{x^2-2x}}\,dx$

29. $\displaystyle\int_2^{3} \frac{2x-3}{\sqrt{4x-x^2}}\,dx$

30. $\displaystyle\int \frac{1}{(x-1)\sqrt{x^2-2x}}\,dx$

31. $\displaystyle\int \frac{x}{x^4+2x^2+2}\,dx$

32. $\displaystyle\int \frac{x}{\sqrt{9+8x^2-x^4}}\,dx$

Think About It **In Exercises 33–36, determine which of the given integrals can be evaluated using the basic integration formulas you have studied so far in the text.**

33. (a) $\displaystyle\int \frac{1}{\sqrt{1-x^2}}\,dx$ (b) $\displaystyle\int \frac{x}{\sqrt{1-x^2}}\,dx$ (c) $\displaystyle\int \frac{1}{x\sqrt{1-x^2}}\,dx$

34. (a) $\displaystyle\int e^{x^2}\,dx$ (b) $\displaystyle\int xe^{x^2}\,dx$ (c) $\displaystyle\int \frac{1}{x^2}e^{1/x}\,dx$

35. (a) $\displaystyle\int \sqrt{x-1}\,dx$ (b) $\displaystyle\int x\sqrt{x-1}\,dx$ (c) $\displaystyle\int \frac{x}{\sqrt{x-1}}\,dx$

36. (a) $\displaystyle\int \frac{1}{1+x^4}\,dx$ (b) $\displaystyle\int \frac{x}{1+x^4}\,dx$ (c) $\displaystyle\int \frac{x^3}{1+x^4}\,dx$

In Exercises 37 and 38, use substitution to evaluate the integral.

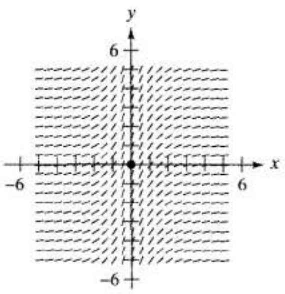
37. $\displaystyle\int \sqrt{e^t-3}\,dt$

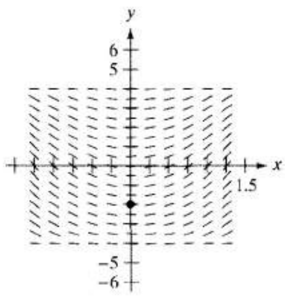
38. $\displaystyle\int \frac{\sqrt{x-2}}{x+1}\,dx$

Direction Fields **In Exercises 39 and 40, a differential equation, a point, and a direction field are given. (a) Sketch two approximate solutions of the differential equation on the direction field, one of which passes through the indicated point. (b) Use integration to find the particular solution of the differential equation and use a graphing utility to graph the solution. Compare the result with the sketches in part (a).**

39. $\dfrac{dy}{dx} = \dfrac{3}{1+x^2}$, $(0, 0)$

40. $\dfrac{dy}{dx} = x\sqrt{16-y^2}$, $(0, -2)$





In Exercises 41 and 42, find the area of the region bounded by the graphs of the equations.

41. $y = \dfrac{1}{x^2-2x+5}$, $y = 0$, $x = 1$, $x = 3$

42. $y = \dfrac{1}{\sqrt{4-x^2}}$, $y = 0$, $x = 0$, $x = 1$

43. ***Approximation*** Determine which value best approximates the area of the region between the x-axis and the function

$$f(x) = \frac{1}{\sqrt{1-x^2}}$$

over the interval $[-0.5, 0.5]$. (Make your selection on the basis of a sketch of the region and *not* by performing any calculations.)

(a) 4 (b) -3 (c) 1 (d) 2 (e) 3

44. ***Approximation*** Sketch the region whose area is represented by the integral

$$\int_0^1 \arcsin x\, dx$$

and use the integration capabilities of a graphing utility to approximate the area.

45. (a) Show that

$$\int_0^1 \frac{4}{1+x^2}\, dx = \pi.$$

(b) Approximate the number π using Simpson's Rule (with $n = 6$) and the integral in part (a).

(c) Approximate the number π by using the integration capabilities of a graphing utility.

46. ***Investigation*** Consider the function

$$F(x) = \frac{1}{2}\int_x^{x+2} \frac{2}{t^2+1}\, dt.$$

(a) Write a short paragraph giving a geometric interpretation of the function $F(x)$ relative to the function

$$f(x) = \frac{2}{x^2+1}.$$

Use what you have written to guess the value of x that will make F maximum.

(b) Perform the specified integration to find an alternative form of $F(x)$. Use calculus to locate the value of x that will make F maximum and compare the result with your guess in part (a).

47. Consider the integral

$$\int \frac{1}{\sqrt{6x - x^2}}\, dx.$$

(a) Evaluate the integral by completing the square of the radicand.

(b) Evaluate the integral by making the substitution $u = \sqrt{x}$.

(c) The antiderivatives in parts (a) and (b) appear significantly different. Use a graphing utility to graph each in the same viewing rectangle and determine the relationship between the two antiderivatives. Find the domain of each.

48. Verify the following rules by differentiating $(a > 0)$.

(a) $\displaystyle\int \frac{du}{\sqrt{a^2 - u^2}} = \arcsin \frac{u}{a} + C$

(b) $\displaystyle\int \frac{du}{a^2 + u^2} = \frac{1}{a} \arctan \frac{u}{a} + C$

(c) $\displaystyle\int \frac{du}{u\sqrt{u^2 - a^2}} = \frac{1}{a} \operatorname{arcsec} \frac{|u|}{a} + C$

49. ***Vertical Motion*** An object is projected upward from ground level with an initial velocity of 500 feet per second. In this exercise, the goal is to analyze the motion of the object during its upward flight.

(a) If air resistance is neglected, find the velocity of the object as a function of time. Use a graphing utility to graph this function.

(b) Use the result in part (a) to find the position function and determine the maximum height attained by the object.

(c) If the air resistance is proportional to the square of the velocity, you obtain the equation

$$\frac{dv}{dt} = -(32 + kv^2)$$

where -32 feet per second per second is the acceleration due to gravity and k is a constant. Find the velocity as a function of time by solving the equation

$$\int \frac{dv}{32 + kv^2} = -\int dt.$$

(d) Use a graphing utility to graph the velocity function $v(t)$ in part (c) if $k = 0.001$. Use the graph to approximate the time t_0 at which the object reaches its maximum height.

(e) Use the integration capabilities of a graphing utility to approximate the integral

$$\int_0^{t_0} v(t)\, dt$$

where $v(t)$ and t_0 are those found in part (d). This is the approximation to the maximum height of the object.

(f) Explain the difference between the results in part (b) and part (e).

FOR FURTHER INFORMATION For more information on this topic, see "What Goes Up Must Come Down; Will Air Resistance Make It Return Sooner, or Later?" by John Lekner in the January 1982 issue of *Mathematics Magazine*.

50. Graph $y_1 = \dfrac{x}{1+x^2}$, $y_2 = \arctan x$, and $y_3 = x$ on $[0, 10]$. Prove that $\dfrac{x}{1+x^2} < \arctan x < x$ for $x > 0$.