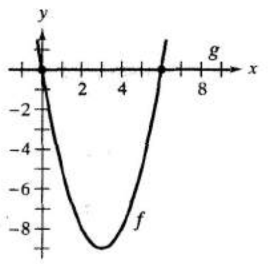
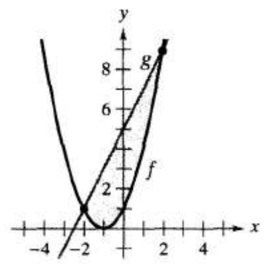
EXERCISES FOR SECTION 6.1

In Exercises 1–6, set up the definite integral that gives the area of the region.

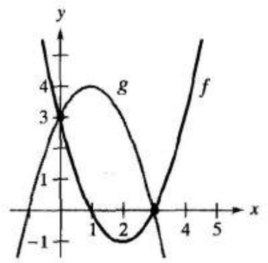
1. $f(x) = x^2 - 6x$
   $g(x) = 0$



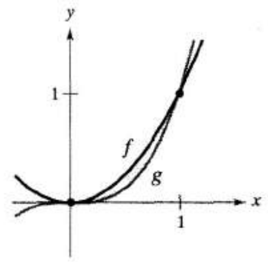
2. $f(x) = x^2 + 2x + 1$
   $g(x) = 2x + 5$



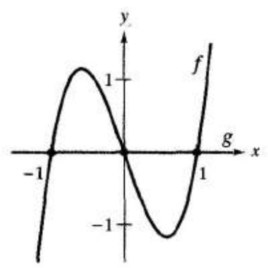
3. $f(x) = x^2 - 4x + 3$
   $g(x) = -x^2 + 2x + 3$



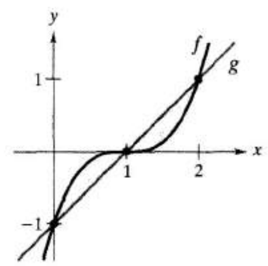
4. $f(x) = x^2$
   $g(x) = x^3$



5. $f(x) = 3(x^3 - x)$
   $g(x) = 0$



6. $f(x) = (x - 1)^3$
   $g(x) = x - 1$



In Exercises 7–10, the integrand of the definite integral is a difference of two functions. Sketch the graph of each function and shade the region whose area is represented by the integral.

7. $\displaystyle\int_0^4 \left[(x + 1) - \frac{x}{2}\right] dx$

8. $\displaystyle\int_{-1}^1 [(1 - x^2) - (x^2 - 1)]\, dx$

9. $\displaystyle\int_0^6 \left[4(2^{-x/3}) - \frac{x}{6}\right] dx$

10. $\displaystyle\int_{-\pi/3}^{\pi/3} (2 - \sec x)\, dx$

Approximation **In Exercises 11 and 12, determine which value best approximates the area of the region bounded by the graphs of *f* and *g*. (Make your selection on the basis of a sketch of the region and *not* by performing any calculations.)**

11. $f(x) = x + 1, \quad g(x) = (x - 1)^2$
    (a) -2 (b) 2 (c) 10 (d) 4 (e) 8

12. $f(x) = 2 - \frac{1}{2}x, \quad g(x) = 2 - \sqrt{x}$
    (a) 1 (b) 6 (c) -3 (d) 3 (e) 4

In Exercises 13–26, sketch the region bounded by the graphs of the algebraic functions and find the area of the region.

13. $f(x) = x^2 - 4x, \; g(x) = 0$
14. $f(x) = 3 - 2x - x^2, \; g(x) = 0$
15. $f(x) = x^2 + 2x + 1, \; g(x) = 3x + 3$
16. $f(x) = -x^2 + 4x + 2, \; g(x) = x + 2$
17. $y = x, \; y = 2 - x, \; y = 0$
18. $y = \dfrac{1}{x^2}, \; y = 0, \; x = 1, \; x = 5$
19. $f(x) = \sqrt{3x} + 1, \; g(x) = x + 1$
20. $f(x) = \sqrt[3]{x}, \; g(x) = x$
21. $f(y) = y^2, \; g(y) = y + 2$
22. $f(y) = y(2 - y), \; g(y) = -y$
23. $f(y) = y^2 + 1, \; g(y) = 0, \; y = -1, \; y = 2$
24. $f(y) = \dfrac{y}{\sqrt{16 - y^2}}, \; g(y) = 0, \; y = 3$
25. $f(x) = \dfrac{4}{x}, \; x = 0, \; y = 1, \; y = 4$
26. $g(x) = \dfrac{4}{2 - x}, \; y = 4, \; x = 0$

In Exercises 27–36, use a graphing utility to graph the region bounded by the graphs of the functions and use the integration capabilities of the graphing utility to find the area of the region.

27. $f(x) = x(x^2 - 3x + 3), \; g(x) = x^2$
28. $f(x) = x^3 - 2x + 1, \; g(x) = -2x, \; x = 1$
29. $y = x^2 - 4x + 3, \; y = 3 + 4x - x^2$
30. $y = x^4 - 2x^2, \; y = 2x^2$
31. $f(x) = x^4 - 4x^2, \; g(x) = x^2 - 4$
32. $f(x) = x^4 - 4x^2, \; g(x) = x^3 - 4x$
33. $f(x) = 1/(1 + x^2), \; g(x) = \frac{1}{2}x^2$
34. $f(x) = 6x/(x^2 + 1), \; y = 0, \; 0 \le x \le 3$

35. $y = \sqrt{1 + x^3}, \; y = \frac{1}{2}x + 2, \; x = 0$

36. $y = x\sqrt{\frac{4 - x}{4 + x}}, \; y = 0, \; x = 4$

In Exercises 37–40, sketch the region bounded by the graphs of the transcendental functions and find the area of the region.

37. $f(x) = 2 \sin x, \; g(x) = \tan x, \; -\frac{\pi}{3} \le x \le \frac{\pi}{3}$

38. $f(x) = \sin 2x, \; g(x) = \cos x, \; \frac{\pi}{6} \le x \le \frac{5\pi}{6}$

39. $f(x) = xe^{-x^2}, \; y = 0, \; 0 \le x \le 1$

40. $f(x) = 3^x, \; g(x) = 2x + 1$

In Exercises 41–44, use a graphing utility to graph the region bounded by the graphs of the functions and use the integration capabilities of the graphing utility to find the area of the region.

41. $f(x) = 2 \sin x + \sin 2x, \; y = 0, \; 0 \le x \le \pi$

42. $f(x) = 2 \sin x + \cos 2x, \; y = 0, \; 0 \le x \le \pi$

43. $f(x) = \frac{1}{x^2}e^{1/x}, \; y = 0, \; 1 \le x \le 3$

44. $g(x) = \frac{4 \ln x}{x}, \; y = 0, \; x = 5$

In Exercises 45 and 46, (a) use a graphing utility to graph the region bounded by the graphs of the equations. (b) Set up the integral giving the area of the region. Can you evaluate the integral by hand? (c) Use the integration capabilities of a graphing utility to approximate the area.

45. $y = \sqrt{\frac{x^3}{4 - x}}, \; y = 0, \; x = 3$

46. $y = \sqrt{x}\, e^x, \; y = 0, \; x = 0, \; x = 1$

In Exercises 47 and 48, use integration to find the area of the triangle having the given vertices.

47. $(0, 0), (a, 0), (b, c)$ 48. $(2, -3), (4, 6), (6, 1)$

In Exercises 49 and 50, set up and evaluate the definite integral that gives the area of the region bounded by the graph of the function and the tangent line to the graph at the indicated point.

49. $f(x) = x^3, (1, 1)$ 50. $f(x) = \frac{1}{x^2 + 1}, \left(1, \frac{1}{2}\right)$

51. ***Think About It*** The graphs of $y = x^4 - 2x^2 + 1$ and $y = 1 - x^2$ intersect at three points. However, the area between the curves *can* be found by a single integral. Explain why this is so, and write an integral for this area.

52. ***Think About It*** The area of the region bounded by the graphs of $y = x^3$ and $y = x$ *cannot* be found by the single integral

$$\int_{-1}^{1} (x^3 - x)\, dx.$$

Explain why this is so. Use symmetry to write a single integral that does represent the area.

In Exercises 53 and 54, find b such that the line $y = b$ divides the region bounded by the graphs of the two equations into two regions of equal area.

53. $y = 9 - x^2, \; y = 0$ 54. $y = 9 - |x|, \; y = 0$

In Exercises 55 and 56, evaluate the limit and sketch the graph of the region whose area is represented by the limit.

55. $\lim_{\|\Delta\| \to 0} \sum_{i=1}^{n} (x_i - x_i^2)\, \Delta x$

where $x_i = i/n$ and $\Delta x = 1/n$

56. $\lim_{\|\Delta\| \to 0} \sum_{i=1}^{n} (4 - x_i^2)\, \Delta x$

where $x_i = -2 + (4i/n)$ and $\Delta x = 4/n$

Revenue **In Exercises 57 and 58, two models R_1 and R_2 are given for revenue (in billions of dollars per year) for a large corporation. The model R_1 gives projected annual revenues from 2000 to 2005, with $t = 0$ corresponding to 2000, and R_2 gives projected revenues if there is a decrease in the rate of growth of corporate sales over the period. Approximate the total reduction in revenue if corporate sales are actually closer to the model R_2.**

57. $R_1 = 7.21 + 0.58t$

$R_2 = 7.21 + 0.45t$

58. $R_1 = 7.21 + 0.26t + 0.02t^2$

$R_2 = 7.21 + 0.1t + 0.01t^2$

59. ***Beef Consumption*** For the years 1985 through 1994, the rate of consumption of beef (in billions of pounds per year) in the United States can be modeled by

$$f(t) = \begin{cases} 27.77 - 0.36t, & 5 \le t \le 10 \\ 21.00 + 0.27t, & 10 \le t \le 14 \end{cases}$$

where t is the time in years, with $t = 5$ corresponding to 1985. *(Source: U.S. Department of Agriculture)*

(a) Use a graphing utility to graph the model.

(b) Suppose the rate of beef consumption for 1990 through 1994 had continued to follow the model for the years 1985 through 1990. How much less beef would have been consumed from 1990 through 1994?