REVIEW EXERCISES FOR CHAPTER 1

In Exercises 1 and 2, determine whether the problem can be solved using precalculus or if calculus is required. If the problem can be solved using precalculus, solve it. If the problem seems to require calculus, explain your reasoning. Use a graphical or numerical approach to estimate the solution.

1. Find the distance between the points $(1, 1)$ and $(3, 9)$ along the curve $y = x^2$.
2. Find the distance between the points $(1, 1)$ and $(3, 9)$ along the line $y = 4x - 3$.

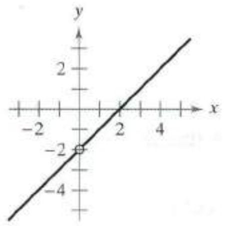
In Exercises 3 and 4, complete the table and use the result to estimate the limit. Use a graphing utility to graph the function to confirm your result.

x	-0.1	-0.01	-0.001	0.001	0.01	0.1
$f(x)$						

3. $\lim_{x \to 0} \dfrac{[1/(x+2)] - (1/2)}{x}$
4. $\lim_{x \to 0} \dfrac{\ln(x+5) - \ln 5}{x}$

In Exercises 5 and 6, use the graph to determine the limit.

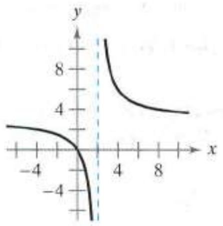
5. $h(x) = \dfrac{x^2 - 2x}{x}$



(a) $\lim_{x \to 0} h(x)$

(b) $\lim_{x \to -1} h(x)$

6. $g(x) = \dfrac{3x}{x - 2}$



(a) $\lim_{x \to 2^+} g(x)$

(b) $\lim_{x \to 0} g(x)$

In Exercises 7–22, find the limit (if it exists).

7. $\lim_{x \to 2} (5x - 3)$
8. $\lim_{x \to 2} (3x + 5)$
9. $\lim_{x \to 2} (5x - 3)(3x + 5)$
10. $\lim_{x \to 2} \dfrac{3x + 5}{5x - 3}$
11. $\lim_{t \to 3} \dfrac{t^2 + 1}{t}$
12. $\lim_{t \to 3} \dfrac{t^2 - 9}{t - 3}$
13. $\lim_{t \to -2} \dfrac{t + 2}{t^2 - 4}$
14. $\lim_{x \to 0} \dfrac{\sqrt{4 + x} - 2}{x}$
15. $\lim_{x \to 0} \dfrac{[1/(x+1)] - 1}{x}$
16. $\lim_{s \to 0} \dfrac{(1/\sqrt{1+s}) - 1}{s}$
17. $\lim_{x \to -5} \dfrac{x^3 + 125}{x + 5}$
18. $\lim_{x \to -2} \dfrac{x^2 - 4}{x^3 + 8}$
19. $\lim_{x \to 0^+} \left(x - \dfrac{1}{x^3}\right)$
20. $\lim_{x \to 2} \dfrac{1}{\sqrt[3]{x^2 - 4}}$
21. $\lim_{\Delta x \to 0} \dfrac{\sin[(\pi/6) + \Delta x] - (1/2)}{\Delta x}$

    [*Hint:* $\sin(\theta + \phi) = \sin\theta\cos\phi + \cos\theta\sin\phi$]
22. $\lim_{\Delta x \to 0} \dfrac{\cos(\pi + \Delta x) + 1}{\Delta x}$

    [*Hint:* $\cos(\theta + \phi) = \cos\theta\cos\phi - \sin\theta\sin\phi$]

In Exercises 23–32, find the one-sided limit.

23. $\lim_{x \to -2^-} \dfrac{2x^2 + x + 1}{x + 2}$
24. $\lim_{x \to (1/2)^+} \dfrac{x}{2x - 1}$
25. $\lim_{x \to -1^+} \dfrac{x + 1}{x^3 + 1}$
26. $\lim_{x \to -1^-} \dfrac{x + 1}{x^4 - 1}$
27. $\lim_{x \to 1^-} \dfrac{x^2 + 2x + 1}{x - 1}$
28. $\lim_{x \to -1^+} \dfrac{x^2 - 2x + 1}{x + 1}$
29. $\lim_{x \to 0^+} \dfrac{\sin 4x}{5x}$
30. $\lim_{x \to 0^+} \dfrac{\sec x}{x}$
31. $\lim_{x \to 0^+} \dfrac{\csc 2x}{x}$
32. $\lim_{x \to 0^-} \dfrac{\cos^2 x}{x}$

Numerical, Graphical, and Analytic Analysis **In Exercises 33 and 34, consider $\lim_{x \to 1^+} f(x)$.**

(a) Complete the table to approximate the limit.

(b) Use a graphing utility to graph the function and use the graph to approximate the limit.

(c) Rationalize the numerator to find the exact value of the limit analytically.

x	1.1	1.01	1.001	1.0001
$f(x)$				

33. $f(x) = \dfrac{\sqrt{2x + 1} - \sqrt{3}}{x - 1}$
34. $f(x) = \dfrac{1 - \sqrt[3]{x}}{x - 1}$

    [*Hint:* $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$]

In Exercises 35–44, determine the intervals on which the function is continuous.

35. $f(x) = [\![x + 3]\!]$
36. $f(x) = \dfrac{3x^2 - x - 2}{x - 1}$
37. $f(x) = \begin{cases} \dfrac{3x^2 - x - 2}{x - 1}, & x \neq 1 \\ 0, & x = 1 \end{cases}$
38. $f(x) = \begin{cases} 5 - x, & x \le 2 \\ 2x - 3, & x > 2 \end{cases}$

39. $f(x) = \dfrac{1}{(x-2)^2}$

40. $f(x) = \sqrt{\dfrac{x+1}{x}}$

41. $f(x) = \dfrac{3}{x+1}$

42. $f(x) = \dfrac{x+1}{2x+2}$

43. $f(x) = \csc\dfrac{\pi x}{2}$

44. $f(x) = \tan 2x$

45. Determine the value of c such that the function is continuous on the entire real line.

$$f(x) = \begin{cases} x+3, & x \le 2 \\ cx+6, & x > 2 \end{cases}$$

46. Determine the values of b and c such that the function is continuous on the entire real line.

$$f(x) = \begin{cases} x+1, & 1 < x < 3 \\ x^2+bx+c, & |x-2| \ge 1 \end{cases}$$

47. ***Compound Interest*** A sum of \$5000 is deposited in a savings plan that pays 12% interest compounded semiannually. The account balance after t years is given by $A = 5000(1.06)^{[\![2t]\!]}$. Use a graphing utility to graph the function and discuss its continuity.

48. ***Cost of Overnight Delivery*** The cost of sending an overnight package from New York to Atlanta is \$9.80 for the first pound and \$2.50 for each additional pound. Use the greatest integer function to create a model for the cost C of overnight delivery of a package weighing x pounds. Use a graphing utility to graph the function and discuss its continuity.

In Exercises 49–52, find the vertical asymptotes (if any) of the function.

49. $g(x) = 1 + \dfrac{2}{x}$

50. $h(x) = \dfrac{4x}{4-x^2}$

51. $f(x) = \dfrac{8}{(x-10)^2}$

52. $f(x) = \csc \pi x$

53. ***Cost of Clean Air*** A utility company burns coal to generate electricity. The cost C in dollars of removing $p\%$ of the air pollutants in the stack emissions is

$$C = \frac{80{,}000p}{100-p}, \qquad 0 \le p < 100.$$

Find the cost of removing (a) 15%, (b) 50%, and (c) 90%. (d) Find the limit of C as $p \to 100^-$.

54. The function f is defined as follows.

$$f(x) = \frac{\tan 2x}{x}, \qquad x \ne 0$$

(a) Find $\displaystyle\lim_{x\to 0} \frac{\tan 2x}{x}$ (if it exists).

(b) Can the function f be defined such that it is continuous at $x = 0$?

Free-Falling Object **In Exercises 55 and 56, use the position function $s(t) = -4.9t^2 + 200$, which gives the height (in meters) of an object that has fallen from a height of 200 meters. The velocity at time $t = a$ seconds is given by**

$$\lim_{t\to a} \frac{s(a)-s(t)}{a-t}.$$

55. Find the velocity of the object when $t = 4$.

56. At what velocity will the object impact the ground?

True or False? **In Exercises 57–63, determine whether the statement is true or false. If it is false, explain why or give an example that shows it is false.**

57. $\displaystyle\lim_{x\to 0} \frac{|x|}{x} = 1$

58. $\displaystyle\lim_{x\to 0} x^3 = 0$

59. If $f(x) = g(x)$ for all real numbers other than $x = 0$, and

$$\lim_{x\to 0} f(x) = L$$

then

$$\lim_{x\to 0} g(x) = L.$$

60. If $\displaystyle\lim_{x\to c} f(x) = L$, then $f(c) = L$.

61. For polynomial functions, the limits from the right and the left must exist and must be equal.

62. $\displaystyle\lim_{x\to 2} f(x) = 3$, where

$$f(x) = \begin{cases} 3, & x \le 2 \\ 0, & x > 2 \end{cases}$$

63. $\displaystyle\lim_{x\to 3} f(x) = 1$, where

$$f(x) = \begin{cases} x-2, & x \le 3 \\ -x^2+8x-14, & x > 3 \end{cases}$$

64. Let $f(x) = \dfrac{x^2-4}{|x-2|}$. Find each limit (if possible).

(a) $\displaystyle\lim_{x\to 2^-} f(x)$

(b) $\displaystyle\lim_{x\to 2^+} f(x)$

(c) $\displaystyle\lim_{x\to 2} f(x)$

65. Let $f(x) = \sqrt{x(x-1)}$.

(a) Find the domain of f.

(b) Find $\displaystyle\lim_{x\to 0^-} f(x)$.

(c) Find $\displaystyle\lim_{x\to 1^+} f(x)$.