LAB SERIES
Lab 2.2

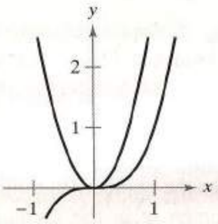
REVIEW EXERCISES FOR CHAPTER 2

In Exercises 1 and 2, find the derivative of the function by using the definition of the derivative.

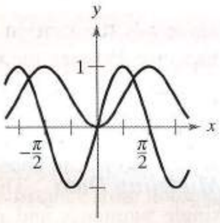
1. $f(x) = x^2 - 2x + 3$
2. $f(x) = \dfrac{x+1}{x-1}$

3. ***Writing*** Each figure shows the graphs of a function and its derivative. Label the graphs as f or f' and write a short paragraph stating the criteria used in making the selection.

(a)



(b)



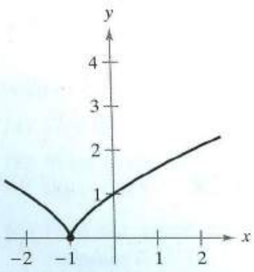
4. ***Writing*** Use the function $f(x) = x\sqrt{4-x}$ for each of the following.
   (a) Use a graphing utility to graph the function.
   (b) Find an equation of the tangent line to the graph of f at the point $(0, 0)$.
   (c) Use a graphing utility to complete the table for $x = 0$.

Δx	$f(x + \Delta x)$	$f(x)$	$\dfrac{f(x+\Delta x) - f(x)}{\Delta x}$
2			
1			
0.5			
0.1			

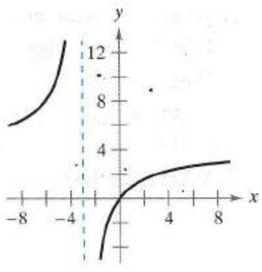
   (d) Write a short paragraph giving the geometric interpretation of the last column of the table. How does it relate to the result in part (b)?

In Exercises 5 and 6, describe the x-values at which f is differentiable.

5. $f(x) = (x+1)^{2/3}$



6. $f(x) = \dfrac{4x}{x+3}$



In Exercises 7–22, find the derivative of the algebraic function.

7. $f(x) = x^3 - 3x^2$
8. $f(x) = x^{1/2} - x^{-1/2}$
9. $f(x) = \dfrac{2x^3 - 1}{x^2}$
10. $f(x) = \dfrac{x+1}{x-1}$
11. $g(t) = \dfrac{2}{3t^2}$
12. $h(x) = \dfrac{2}{(3x)^2}$
13. $f(x) = \sqrt{1 - x^3}$
14. $f(x) = \sqrt[3]{x^2 - 1}$
15. $f(x) = (3x^2 + 7)(x^2 - 2x + 3)$
16. $f(s) = (s^2 - 1)^{5/2}(s^3 + 5)$
17. $f(x) = \left(x^2 + \dfrac{1}{x}\right)^5$
18. $h(\theta) = \dfrac{\theta}{(1 - \theta)^3}$
19. $f(x) = \dfrac{x^2 + x - 1}{x^2 - 1}$
20. $f(x) = \dfrac{6x - 5}{x^2 + 1}$
21. $f(x) = \dfrac{1}{4 - 3x^2}$
22. $f(x) = \dfrac{9}{3x^2 - 2x}$

In Exercises 23–34, find the derivative of the trigonometric function.

23. $y = 3\cos(3x + 1)$
24. $y = 1 - \cos 2x + 2\cos^2 x$
25. $y = \frac{1}{2}\csc 2x$
26. $y = \csc 3x + \cot 3x$
27. $y = \dfrac{x}{2} - \dfrac{\sin 2x}{4}$
28. $y = \dfrac{1 + \sin x}{1 - \sin x}$
29. $y = \frac{2}{3}\sin^{3/2}x - \frac{2}{7}\sin^{7/2}x$
30. $y = \dfrac{\sec^7 x}{7} - \dfrac{\sec^5 x}{5}$
31. $y = -x\tan x$
32. $y = x\cos x - \sin x$
33. $y = \dfrac{\sin x}{x^2}$
34. $y = \dfrac{\cos(x - 1)}{x - 1}$

In Exercises 35–42, use a symbolic differentiation utility to find the derivative of the function. Use the utility to graph the function and its derivative on the same set of coordinate axes. Describe the behavior of the function that corresponds to any zeros of the graph of the derivative.

35. $f(t) = t^2(t - 1)^5$
36. $f(x) = [(x - 2)(x + 4)]^2$
37. $g(x) = \dfrac{2x}{\sqrt{x + 1}}$
38. $g(x) = x\sqrt{x^2 + 1}$
39. $f(t) = \sqrt{t + 1}\,\sqrt[3]{t + 1}$
40. $y = \sqrt{3x}(x + 2)^3$
41. $y = \tan\sqrt{1 - x}$
42. $y = 2\csc^3(\sqrt{x})$

In Exercises 43 and 44, find the value of the derivative of the function at the indicated point. Use the derivative feature of a graphing utility to confirm your result.

Function	*Point*
43. $f(x) = \cos\dfrac{\pi x}{2}$	$\left(\frac{2}{3}, \frac{1}{2}\right)$
44. $f(x) = x + \dfrac{8}{x^2}$	$(2, 4)$

In Exercises 45–48, find the second derivative of the function.

45. $y = 2x^2 + \sin 2x$

46. $y = \dfrac{1}{x} + \tan x$

47. $f(x) = \cot x$

48. $y = \sin^2 x$

In Exercises 49–52, use a symbolic differentiation utility to find the second derivative of the function.

49. $f(t) = \dfrac{t}{(1-t)^2}$

50. $g(x) = \dfrac{6x-5}{x^2+1}$

51. $g(x) = x \tan x$

52. $h(x) = x\sqrt{x^2-1}$

In Exercises 53–58, use implicit differentiation to find dy/dx.

53. $x^2 + 3xy + y^3 = 10$

54. $x^2 + 9y^2 - 4x + 3y = 0$

55. $y\sqrt{x} - x\sqrt{y} = 16$

56. $y^2 = (x-y)(x^2+y)$

57. $x \sin y = y \cos x$

58. $\cos(x+y) = x$

In Exercises 59–64, find the equations of the tangent line and the normal line to the graph of the equation at the indicated point. Use a graphing utility to graph the equation, the tangent line, and the normal line.

59. $y = (x+3)^3, \quad (-2, 1)$

60. $y = (x-2)^2, \quad (2, 0)$

61. $x^2 + y^2 = 20, \quad (2, 4)$

62. $x^2 - y^2 = 16, \quad (5, 3)$

63. $y = \sqrt[3]{(x-2)^2}, \quad (3, 1)$

64. $y = \dfrac{2x}{1-x^2}, \quad (0, 0)$

65. Find the points on the graph of $f(x) = \frac{1}{3}x^3 + x^2 - x - 1$ when the slope is (a) -1, (b) 2, and (c) 0.

66. Find the points on the graph of $f(x) = x^2 + 1$ when the slope is (a) -1, (b) 0, and (c) 1.

67. Sketch the graph of $f(x) = 4 - |x-2|$.
   (a) Is f continuous at $x = 2$?
   (b) Is f differentiable at $x = 2$? Explain.

68. Sketch the graph of

$$f(x) = \begin{cases} x^2 + 4x + 2, & x < -2 \\ 1 - 4x - x^2, & x \geq -2. \end{cases}$$

   (a) Is f continuous at $x = -2$?
   (b) Is f differentiable at $x = -2$? Explain.

In Exercises 69 and 70, show that the function satisfies the equation.

Function	*Equation*
69. $y = 2\sin x + 3\cos x$	$y'' + y = 0$
70. $y = \dfrac{10 - \cos x}{x}$	$xy' + y = \sin x$

71. ***Refrigeration*** The temperature T of food put in a freezer is

$$T = \frac{700}{t^2 + 4t + 10}$$

where t is the time in hours. Find the rate of change T with respect to t at each of the following times.
(a) $t = 1$ (b) $t = 3$ (c) $t = 5$ (d) $t = 10$

72. ***Fluid Flow*** The emergent velocity v of a liquid flowing from a hole in the bottom of a tank is given by $v = \sqrt{2gh}$, where g is the acceleration due to gravity (32 feet per second per second) and h is the depth of the liquid in the tank. Find the rate of change of v with respect to h when (a) $h = 9$ and (b) $h = 4$. (Note that $g = +32$ feet per second per second. The sign of g depends on how a problem is modeled. In this case, letting g be negative would produce an imaginary value for v.)

73. ***Vibrating String*** When a guitar string is plucked, it vibrates with a frequency of $F = 200\sqrt{T}$, where F is measured in vibrations per second and the tension T is measured in pounds. Find the rate of change in F when (a) $T = 4$ and (b) $T = 9$.

74. ***Vertical Motion*** A ball is dropped from a height of 100 feet. One second later, another ball is dropped from a height of 75 feet. Which ball hits the ground first?

75. ***Vertical Motion*** What is the smallest initial velocity that is required to throw a stone 49 feet up to the top of a silo?

76. ***Vertical Motion*** A bomb is dropped from an airplane at an altitude of 14,400 feet. How long will it take to reach the ground? (Because of the motion of the plane, the fall will not be vertical, but the time will be the same as that for a vertical fall.) The plane is moving at 600 miles per hour. How far will the bomb move horizontally after it is released from the plane?

77. ***Projectile Motion*** A ball thrown follows a path described by $y = x - 0.02x^2$.
   (a) Sketch a graph of the path.
   (b) Find the total horizontal distance the ball was thrown.
   (c) At what x-value does the ball reach its maximum height? (Use the symmetry of the path.)
   (d) Find an equation that gives the instantaneous rate of change of the height of the ball with respect to the horizontal change. Evaluate the equation at $x = 0$, 10, 25, 30, and 50.
   (e) What is the instantaneous rate of change of the height when the ball reaches its maximum height?

78. A point moves along the curve $y = \sqrt{x}$ in such a way that the y-value is increasing at a rate of 2 units per second. At what rate is x changing for each of the following values?
(a) $x = \frac{1}{2}$ (b) $x = 1$ (c) $x = 4$