REVIEW EXERCISES FOR CHAPTER 3

1. ***Think About It*** Give the definition of a critical number, and graph an arbitrary function f showing the different types of critical numbers.

2. Consider the odd function f that is continuous, is differentiable, and has the functional values shown in the table.

x	-5	-4	-1	0	2	3	6
$f(x)$	1	3	2	0	-1	-4	0

(a) Determine $f(4)$.

(b) Determine $f(-3)$.

(c) Plot the points and make a possible sketch of the graph of f on the interval $[-6, 6]$. What is the smallest number of critical points in the interval? Explain.

(d) Does there exist at least one real number c in the interval $(-6, 6)$ where $f'(c) = -1$? Explain.

(e) Is it possible that $\lim_{x \to 0} f(x)$ does not exist? Explain.

(f) Is it necessary that $f'(x)$ exists at $x = 2$? Explain.

In Exercises 3 and 4, locate the absolute extrema of the function on the closed interval. Use a graphing utility to graph the function over the indicated interval to confirm your results.

3. $g(x) = 2x + 5 \cos x, \quad [0, 2\pi]$

4. $f(x) = \dfrac{x}{\sqrt{x^2 + 1}}, \quad [0, 2]$

5. ***Think About It*** Consider the function $f(x) = 3 - |x - 4|$.

(a) Graph the function and verify that $f(1) = f(7)$.

(b) Note that $f'(x)$ is not equal to zero for any x in $[1, 7]$. Explain why this does not contradict Rolle's Theorem.

6. ***Think About It*** Can the Mean Value Theorem be applied to the function $f(x) = 1/x^2$ on the interval $[-2, 1]$? Explain.

In Exercises 7–10, find the point(s) guaranteed by the Mean Value Theorem for the indicated interval.

7. $f(x) = x^{2/3}$
$1 \le x \le 8$

8. $f(x) = \dfrac{1}{x}$
$1 \le x \le 4$

9. $f(x) = x - \cos x$
$-\dfrac{\pi}{2} \le x \le \dfrac{\pi}{2}$

10. $f(x) = \sqrt{x} - 2x$
$0 \le x \le 4$

11. For the function $f(x) = Ax^2 + Bx + C$, determine the value of c guaranteed by the Mean Value Theorem on the interval $[x_1, x_2]$.

12. Demonstrate the result of Exercise 11 for $f(x) = 2x^2 - 3x + 1$ on the interval $[0, 4]$.

In Exercises 13–16, find the critical numbers (if any) and the open intervals on which the function is increasing or decreasing.

13. $f(x) = (x - 1)^2(x - 3)$

14. $g(x) = (x + 1)^3$

15. $h(x) = \sqrt{x}(x - 3)$

16. $f(x) = \sin x + \cos x, \quad 0 \le x \le 2\pi$

In Exercises 17 and 18, use the First Derivative Test to find any relative extrema of the function. Use a graphing utility to verify your results.

17. $h(t) = \dfrac{1}{4}t^4 - 8t$

18. $g(x) = \dfrac{3}{2}\sin\left(\dfrac{\pi x}{2} - 1\right), \quad [0, 4]$

In Exercises 19 and 20, find any points of inflection of the function.

19. $f(x) = x + \cos x$
$0 \le x \le 2\pi$

20. $f(x) = (x + 2)^2(x - 4)$

In Exercises 21–24, find the limit.

21. $\lim_{x \to \infty} \dfrac{2x^2}{3x^2 + 5}$

22. $\lim_{x \to \infty} \dfrac{2x}{3x^2 + 5}$

23. $\lim_{x \to \infty} \dfrac{5 \cos x}{x}$

24. $\lim_{x \to \infty} \dfrac{3x}{\sqrt{x^2 + 4}}$

In Exercises 25–28, find any vertical and horizontal asymptotes of the graph of the function. Use a graphing utility to verify your results.

25. $h(x) = \dfrac{2x + 3}{x - 4}$

26. $g(x) = \dfrac{5x^2}{x^2 + 2}$

27. $f(x) = \dfrac{3}{x} - 2$

28. $f(x) = \dfrac{3x}{\sqrt{x^2 + 2}}$

In Exercises 29–32, use a graphing utility to graph the function. Use the graph to approximate any relative extrema or asymptotes.

29. $f(x) = x^3 + \dfrac{243}{x}$

30. $f(x) = |x^3 - 3x^2 + 2x|$

31. $f(x) = \dfrac{x - 1}{1 + 3x^2}$

32. $g(x) = \dfrac{\pi^2}{3} - 4 \cos x + \cos 2x$

In Exercises 33–50, make use of domain, range, symmetry, asymptotes, intercepts, relative extrema, and/or points of inflection to obtain an accurate graph of the function.

33. $f(x) = 4x - x^2$
34. $f(x) = 4x^3 - x^4$
35. $f(x) = x\sqrt{16 - x^2}$
36. $f(x) = (x^2 - 4)^2$
37. $f(x) = (x - 1)^3(x - 3)^2$
38. $f(x) = (x - 3)(x + 2)^3$
39. $f(x) = x^{1/3}(x + 3)^{2/3}$
40. $f(x) = (x - 2)^{1/3}(x + 1)^{2/3}$
41. $f(x) = \dfrac{x + 1}{x - 1}$
42. $f(x) = \dfrac{2x}{1 + x^2}$
43. $f(x) = \dfrac{4}{1 + x^2}$
44. $f(x) = \dfrac{x^2}{1 + x^4}$
45. $f(x) = x^3 + x + \dfrac{4}{x}$
46. $f(x) = x^2 + \dfrac{1}{x}$
47. $f(x) = |x^2 - 9|$
48. $f(x) = |x - 1| + |x - 3|$
49. $f(x) = x + \cos x, \quad 0 \le x \le 2\pi$
50. $f(x) = \dfrac{1}{\pi}(2 \sin \pi x - \sin 2\pi x), \quad -1 \le x \le 1$

51. ***Minimum Distance*** At noon, ship A is 100 kilometers due east of ship B. Ship A is sailing west at 12 kilometers per hour, and ship B is sailing south at 10 kilometers per hour. At what time will the ships be nearest to each other, and what will this distance be?

52. ***Maximum Area*** Find the dimensions of the rectangle of maximum area, with sides parallel to the coordinate axes, that can be inscribed in the ellipse given by

$$\frac{x^2}{144} + \frac{y^2}{16} = 1.$$

53. ***Minimum Length*** A right triangle in the first quadrant has the coordinate axes as sides, and the hypotenuse passes through the point (1, 8). Find the vertices of the triangle such that the length of the hypotenuse is minimum.

54. ***Minimum Length*** The wall of a building is to be braced by a beam that must pass over a parallel fence 5 feet high and 4 feet from the building. Find the length of the shortest beam that can be used.

55. ***Maximum Area*** Three sides of a trapezoid have the same length s. Of all such possible trapezoids, show that the one of maximum area has a fourth side of length $2s$.

56. ***Maximum Area*** Show that the greatest area of any rectangle inscribed in a triangle is one-half that of the triangle.

57. ***Minimum Distance*** Find the length of the longest pipe that can be carried level around a right-angle corner at the intersection of two corridors of widths 4 feet and 6 feet. (Do not use trigonometry.)

58. ***Minimum Distance*** Rework Exercise 57, given corridors of widths a meters and b meters.

59. ***Minimum Distance*** A hallway of width 6 feet meets a hallway of width 9 feet at right angles. Find the length of the longest pipe that can be carried level around this corner. [*Hint:* If L is the length of the pipe, show that

$$L = 6 \csc \theta + 9 \csc\left(\frac{\pi}{2} - \theta\right)$$

where θ is the angle between the pipe and the wall of the narrower hallway.]

60. ***Minimum Distance*** Rework Exercise 59, given that one hallway is of width a meters and the other is of width b meters. Show that the result is the same as in Exercise 58.

61. ***Harmonic Motion*** The height of an object attached to a spring is given by the harmonic equation

$$y = \tfrac{1}{3} \cos 12t - \tfrac{1}{4} \sin 12t$$

where y is measured in inches and t is measured in seconds.
   (a) Calculate the height and velocity of the object when $t = \pi/8$ second.
   (b) Show that the maximum displacement of the object is $\frac{5}{12}$ inch.
   (c) Find the period P of y. Also, find the frequency f (number of oscillations per second) if $f = 1/P$.

62. ***Writing*** The general equation giving the height of an oscillating object attached to a spring is

$$y = A \sin\sqrt{\frac{k}{m}}t + B \cos\sqrt{\frac{k}{m}}t$$

where k is the spring constant and m is the mass of the object.
   (a) Show that the maximum displacement of the object is $\sqrt{A^2 + B^2}$.
   (b) Show that the object oscillates with a frequency of
   $$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}.$$
   (c) How is the frequency changed if the stiffness k of the spring is increased?
   (d) How is the frequency changed if the mass m of the object is increased?

In Exercises 63 and 64, use Newton's Method to approximate any real zeros of the function accurate to three decimal places. Use the root-finding capabilities of a graphing utility to verify your results.

63. $f(x) = x^3 - 3x - 1$
64. $f(x) = x^3 + 2x + 1$

In Exercises 65 and 66, use Newton's Method to approximate, to three decimal places, the x-value of the points of intersection of the equations. Use a graphing utility to verify your results.

65. $y = x^4$
    $y = x + 3$
66. $y = \sin \pi x$
    $y = 1 - x$

In Exercises 67 and 68, find the differential dy.

67. $y = x(1 - \cos x)$ **68.** $y = \sqrt{36 - x^2}$

69. ***Modeling Data*** Outlays for national defense S (in billions of dollars) for the years 1970 through 1995 are given in the table, where t is the time in years, with $t = 0$ corresponding to 1970. *(Source: U.S. Office of Management and Budget)*

t	0	1	2	3	4	5	6
S	81.7	78.9	79.2	76.7	79.3	86.5	89.6

t	7	8	9	10	11	12	13
S	97.2	104.5	116.3	134.0	157.5	185.3	209.9

t	14	15	16	17	18	19	20
S	227.4	252.7	273.4	282.0	290.4	303.6	299.3

t	21	22	23	24	25
S	273.3	298.4	291.1	281.6	271.6

(a) Use the regression capabilities of a graphing utility to find a model of the form $S = at^3 + bt^2 + ct + d$ for the data.

(b) Use a graphing utility to plot the data and graph the model.

(c) For the years given in the table, when does the model indicate that the outlay for national defense is at a maximum?

(d) For the years given in the table, when does the model indicate that the outlay for national defense is increasing at the greatest rate?

70. ***Modeling Data*** Petroleum imports I (in billions of dollars) for the years 1980 through 1994 are given in the table, where t is the time in years, with $t = 0$ corresponding to 1980. *(Source: "Highlights of U.S. Export and Import Trade" and the U.S. Bureau of Census)*

t	0	1	2	3	4
I	77.6	75.6	59.4	52.3	55.9

t	5	6	7	8	9
I	49.6	34.1	41.5	38.8	49.1

t	10	11	12	13	14
I	60.5	50.1	50.4	49.7	49.6

(a) Use the regression capabilities of a graphing utility to find a model of the form

$$I = at^4 + bt^3 + ct^2 + dt + e$$

for the data.

(b) Use a graphing utility to plot the data and graph the model.

(c) For the years given in the table, when does the model indicate that the cost of petroleum imports is at a minimum?

(d) For the years given in the table, when does the model indicate the cost of petroleum imports is decreasing at the greatest rate?

71. ***Surface Area and Volume*** The diameter of a sphere is measured to be 18 centimeters with a maximum possible error of 0.05 centimeter. Use the differential to approximate the possible propagated error and percent error in calculating the surface area and the volume of the sphere.

72. ***Demand Function*** A company finds that the demand for its commodity is

$$p = 75 - \tfrac{1}{4}x.$$

If x changes from 7 to 8, find and compare the values of Δp and dp.

73. ***Profit*** Find the maximum profit if the demand equation is $p = 36 - 4x$ and the total cost is $C = 2x^2 + 6$.

74. ***Profit*** The cost C of producing x units per day is

$$C = \tfrac{1}{4}x^2 + 62x + 125$$

and the price p per unit is

$$p = 75 - \tfrac{1}{3}x.$$

(a) What daily output produces maximum profit?

(b) What daily output produces minimum average cost?

(c) Find the elasticity of demand.

75. ***Revenue*** For groups of 80 or more, a charter bus company determines the rate per person according to the following formula.

$$\text{Rate} = \$8.00 - \$0.05(n - 80), \qquad n \ge 80$$

What number of passengers will give the bus company maximum revenue?

76. ***Cost*** The cost of fuel for running a locomotive is proportional to the $\frac{3}{2}$ power of the speed $(s^{3/2})$ and is \$50 per hour for a speed of 25 miles per hour. Other fixed costs amount to an average of \$100 per hour. Find the speed that will minimize the cost per mile.

77. ***Inventory Cost*** The cost of inventory depends on the ordering and storage costs according to the inventory model

$$C = \left(\frac{Q}{x}\right)s + \left(\frac{x}{2}\right)r.$$

Determine the order size that will minimize the cost, assuming that sales occur at a constant rate, Q is the number of units sold per year, r is the cost of storing one unit for 1 year, s is the cost of placing an order, and x is the number of units per order.

78. ***Taxes*** The demand and cost equations for a certain product are $p = 600 - 3x$ and $C = 0.3x^2 + 6x + 600$, where p is the price per unit, x is the number of units, and C is the cost of producing x units. If t is the excise tax per unit, the profit for producing x units is $P = xp - C - xt$. Find the maximum profit for

(a) $t = 5$, (b) $t = 10$, and (c) $t = 20$.

79. Find the maximum and minimum points on the graph of

$$x^2 + 4y^2 - 2x - 16y + 13 = 0$$

(a) without using calculus.

(b) using calculus.

80. Consider the function $f(x) = x^n$ for positive integer values of n.

(a) For what values of n does the function have a relative minimum at the origin?

(b) For what values of n does the function have a point of inflection at the origin?

True or False? **In Exercises 81 and 82, determine whether the statement is true or false. If it is false, explain why or give an example that shows it is false.**

81. If c is a critical number of f, then f has a relative extremum at $x = c$.

82. The function $y = f(x)$ can have at most one horizontal asymptote.

83. ***Writing*** A newspaper headline states that "The rate of growth of the national deficit is decreasing." What does this mean? What does it imply about the graph of the deficit as a function of time?

84. ***Modeling Data*** The manager of a store recorded the annual sales S (in thousands of dollars) of a product over a period of 7 years, as shown in the table, where t is the time in years, with $t = 1$ corresponding to 1991.

t	1	2	3	4	5	6	7
S	5.4	6.9	11.5	15.5	19.0	22.0	23.6

(a) Use the regression capabilities of a graphing utility to find a model of the form $S = at^3 + bt^2 + ct + d$ for the data.

(b) Use a graphing utility to plot the data and graph the model.

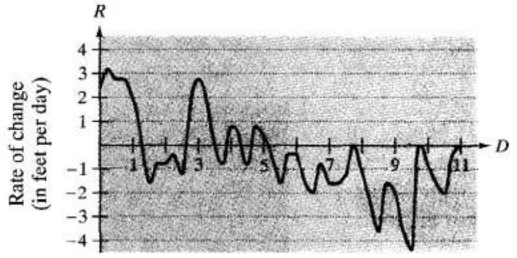
(c) Use calculus to find the time t when sales were increasing at the greatest rate.

(d) Do you think the model would be accurate for predicting future sales? Explain.

SECTION PROJECT

Whenever the Connecticut River reaches a level of 105 feet above sea level, two Northampton, Massachusetts flood control station operators begin a round-the-clock river watch. Every two hours, they check the height of the river, using a scale marked off in tenths of a foot, and record the data in a log book. In the spring of 1996, the flood watch lasted from April 4, when the river reached 105 feet and was rising at 0.2 foot per hour, until April 25, when the level subsided again to 105 feet. Between those dates, their log shows that the river rose and fell several times, at one point coming close to the 115-foot mark. If the river had reached 115 feet, the city would have closed down Mount Tom Road (Route 5, south of Northampton).

The graph below shows the *rate of change* of the level of the river during one portion of the flood watch. Use the graph to answer the following questions.



Day (0 ↔ 12:01AM April 14)

(a) On what date was the river rising most rapidly? How do you know?

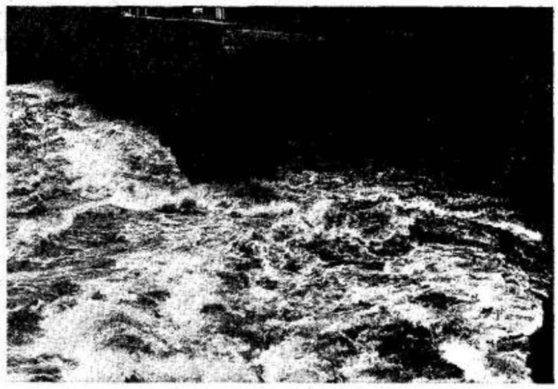
(b) On what date was the river falling most rapidly? How do you know?

(c) There were two dates in a row on which the river rose, then fell, then rose again during the course of the day. On which days did this occur, and how do you know?

(d) At one minute past midnight, April 14, the river level was 111.0 feet. Estimate its height 24 hours later and 48 hours later. Explain how you made your estimates.

(e) The river crested at 114.4 feet. On what date do you think this occurred?

(Submitted by Mary Murphy, Smith College, Northampton, MA)



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