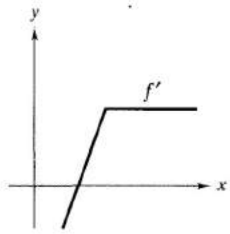
REVIEW EXERCISES FOR CHAPTER 4

In Exercises 1 and 2, use the graph of f' to sketch a graph of f.

1.


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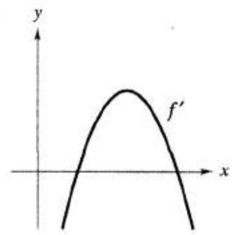


In Exercises 3–8, find the indefinite integral.

3. $\int (2x^2 + x - 1)\, dx$

4. $\int \frac{2}{\sqrt[3]{3x}}\, dx$

5. $\int \frac{x^3 + 1}{x^2}\, dx$

6. $\int \frac{x^3 - 2x^2 + 1}{x^2}\, dx$

7. $\int (4x - 3 \sin x)\, dx$

8. $\int (5 \cos x - 2 \sec^2 x)\, dx$

9. Find the particular solution of the differential equation $f'(x) = -2x$ whose graph passes through the point $(-1, 1)$.

10. Find the particular solution of the differential equation $f''(x) = 6(x - 1)$ whose graph passes through the point $(2, 1)$ and at that point is tangent to the line $3x - y - 5 = 0$.

11. ***Velocity and Acceleration*** An airplane taking off from a runway travels 3600 feet before lifting off. If it starts from rest, moves with constant acceleration, and makes the run in 30 seconds, with what speed does it lift off?

12. ***Velocity and Acceleration*** The speed of a car traveling in a straight line is reduced from 45 to 30 miles per hour in a distance of 264 feet. Find the distance in which the car can be brought to rest from 30 miles per hour, assuming the same constant deceleration.

13. ***Velocity and Acceleration*** A ball is thrown vertically upward from ground level with an initial velocity of 96 feet per second.
    (a) How long will it take the ball to rise to its maximum height?
    (b) What is the maximum height?
    (c) When is the velocity of the ball one-half the initial velocity?
    (d) What is the height of the ball when its velocity is one-half the initial velocity?

14. ***Velocity and Acceleration*** Repeat Exercise 13 for an initial velocity of 40 meters per second.

15. Write in the sigma notation (a) the sum of the first ten positive odd integers, (b) the sum of the cubes of the first n positive integers, and (c) $6 + 10 + 14 + 18 + \cdots + 42$.

16. Evaluate each of the following sums for $x_1 = 2, x_2 = -1, x_3 = 5, x_4 = 3$, and $x_5 = 7$.
    (a) $\frac{1}{5}\sum_{i=1}^{5} x_i$ (b) $\sum_{i=1}^{5} \frac{1}{x_i}$
    (c) $\sum_{i=1}^{5} (2x_i - x_i^2)$ (d) $\sum_{i=2}^{5} (x_i - x_{i-1})$

17. Consider the region bounded by $y = mx, y = 0, x = 0$, and $x = b$.
    (a) Find the upper and lower sums to approximate the area of the region when $\Delta x = b/4$.
    (b) Find the upper and lower sums to approximate the area of the region when $\Delta x = b/n$.
    (c) Find the area of the region by letting n approach infinity in both sums in part (b). Show that in each case you obtain the formula for the area of a triangle.
    (d) Find the area of the region using the Fundamental Theorem of Calculus.

18. (a) Find the area of the region bounded by the graphs of $y = x^3, y = 0, x = 1$, and $x = 3$ by the limit definition.
    (b) Find the area of the region using the Fundamental Theorem of Calculus.

In Exercises 19 and 20, use the given values to find the value of each definite integral.

19. If $\int_2^6 f(x)\, dx = 10$ and $\int_2^6 g(x)\, dx = 3$, find
    (a) $\int_2^6 [f(x) + g(x)]\, dx$. (b) $\int_2^6 [f(x) - g(x)]\, dx$.
    (c) $\int_2^6 [2f(x) - 3g(x)]\, dx$. (d) $\int_2^6 5f(x)\, dx$.

20. If $\int_0^3 f(x)\, dx = 4$ and $\int_3^6 f(x)\, dx = -1$, find
    (a) $\int_0^6 f(x)\, dx$. (b) $\int_6^3 f(x)\, dx$.
    (c) $\int_4^4 f(x)\, dx$. (d) $\int_3^6 -10f(x)\, dx$.

In Exercises 21–34, find the indefinite integral.

21. $\int (x^2 + 1)^3\, dx$

22. $\int \left(x + \frac{1}{x}\right)^2 dx$

23. $\int \frac{x^2}{\sqrt{x^3 + 3}}\, dx$

24. $\int x^2\sqrt{x^3 + 3}\, dx$

25. $\int x(1 - 3x^2)^4\, dx$

26. $\int \frac{x + 3}{(x^2 + 6x - 5)^2}\, dx$

27. $\int \sin^3 x \cos x\, dx$

28. $\int x \sin 3x^2\, dx$

29. $\int \frac{\sin\theta}{\sqrt{1-\cos\theta}}\, d\theta$

30. $\int \frac{\cos x}{\sqrt{\sin x}}\, dx$

31. $\int \tan^n x \sec^2 x\, dx, \quad n \neq -1$

32. $\int \sec 2x \tan 2x\, dx$

33. $\int (1 + \sec \pi x)^2 \sec \pi x \tan \pi x\, dx$

34. $\int \cot^4 \alpha \csc^2 \alpha\, d\alpha$

In Exercises 35–46, use the Fundamental Theorem of Calculus to evaluate the definite integral. Use a graphing utility to verify your result.

35. $\int_0^4 (2 + x)\, dx$

36. $\int_{-1}^1 (t^2 + 2)\, dt$

37. $\int_{-1}^1 (4t^3 - 2t)\, dt$

38. $\int_3^6 \frac{x}{3\sqrt{x^2 - 8}}\, dx$

39. $\int_0^3 \frac{1}{\sqrt{1+x}}\, dx$

40. $\int_0^1 x^2(x^3 + 1)^3\, dx$

41. $\int_4^9 x\sqrt{x}\, dx$

42. $\int_1^2 \left(\frac{1}{x^2} - \frac{1}{x^3}\right) dx$

43. $2\pi \int_0^1 (y + 1)\sqrt{1 - y}\, dy$

44. $2\pi \int_{-1}^0 x^2\sqrt{x + 1}\, dx$

45. $\int_0^\pi \cos \frac{x}{2}\, dx$

46. $\int_{-\pi/4}^{\pi/4} \sin 2x\, dx$

In Exercises 47–52, sketch the graph of the region whose area is given by the integral and find the area.

47. $\int_1^3 (2x - 1)\, dx$

48. $\int_0^2 (x + 4)\, dx$

49. $\int_3^4 (x^2 - 9)\, dx$

50. $\int_{-1}^2 (-x^2 + x + 2)\, dx$

51. $\int_0^1 (x - x^3)\, dx$

52. $\int_0^1 \sqrt{x}(1 - x)\, dx$

In Exercises 53–56, sketch the region bounded by the graphs of the equations and determine its area.

53. $y = \frac{4}{\sqrt{x+1}}, \; y = 0, \; x = 0, \; x = 8$

54. $y = x, \; y = x^5$

55. $y = \sec^2 x, \; y = 0, \; x = 0, \; x = \frac{\pi}{3}$

56. $y = \cos x, \; y = 0, \; x = -\frac{\pi}{4}, \; x = \frac{\pi}{4}$

In Exercises 57–60, find the average value of the function over the interval. Find the values of x where the function assumes its average value and graph the function.

Function	*Interval*
57. $f(x) = \frac{1}{\sqrt{x-1}}$	$[5, 10]$
58. $f(x) = x^3$	$[0, 2]$
59. $f(x) = x$	$[0, 4]$
60. $f(x) = x^2 - \frac{1}{x^2}$	$[1, 2]$

In Exercises 61 and 62, use the Trapezoidal Rule and Simpson's Rule with $n = 4$, and use the integration capabilities of a graphing utility, to approximate the definite integral. Compare the results.

61. $\int_1^2 \frac{1}{1 + x^3}\, dx$

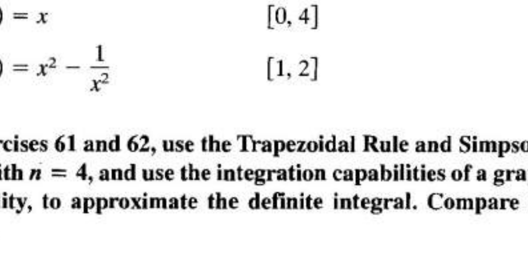
62. $\int_0^1 \frac{x^{3/2}}{3 - x^2}\, dx$

63. ***Air Conditioning Costs*** The temperature in degrees Fahrenheit is $T = 72 + 12 \sin[\pi(t - 8)/12]$ where t is time in hours, with $t = 0$ representing midnight. Suppose the hourly cost of cooling a house is \$0.10 per degree.

(a) Find the cost C of cooling the house if its thermostat is set at 72°F by evaluating the integral

$$C = 0.1\int_8^{20} \left[72 + 12 \sin \frac{\pi(t-8)}{12} - 72\right] dt.$$

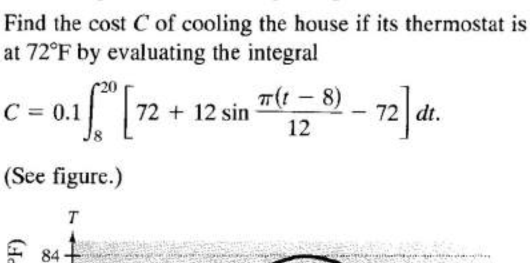
(See figure.)



(b) Find the savings from resetting the thermostat to 78°F by evaluating the integral

$$C = 0.1\int_{10}^{18} \left[72 + 12 \sin \frac{\pi(t-8)}{12} - 78\right] dt.$$

(See figure.)



64. ***Production Scheduling*** A manufacturer of fertilizer finds that national sales of fertilizer follow the seasonal pattern

$$F = 100{,}000\left[1 + \sin\frac{2\pi(t - 60)}{365}\right]$$

where F is measured in pounds and t is time in days, with $t = 1$ representing January 1. The manufacturer wants to set up a schedule to produce a uniform amount each day. What should this amount be?

65. ***Respiratory Cycle*** For a person at rest, the rate of air intake v, in liters per second, during a respiratory cycle is

$$v = 0.85\sin\frac{\pi t}{3}$$

where t is time in seconds. Find the volume, in liters, of air inhaled during one cycle by integrating the function over the interval $[0, 3]$.

66. ***Respiratory Cycle*** After exercising a few minutes, a person has a respiratory cycle for which the rate of air intake is

$$v = 1.75\sin\frac{\pi t}{2}.$$

Find the volume, in liters, of air inhaled during one cycle by integrating the function over the interval $[0, 2]$. How much does the person's lung capacity increase as a result of exercising? (Compare your answer with that found in Exercise 65.)

67. Suppose that gasoline is increasing in price according to the equation

$$p = 1.20 + 0.04t$$

where p is the dollar price per gallon and $t = 0$ represents the year 1990. If an automobile is driven 15,000 miles a year and gets M miles per gallon, the annual fuel cost is

$$C = \frac{15{,}000}{M}\int_t^{t+1} p\,ds.$$

Estimate the annual fuel cost for the years (a) 2000 and (b) 2005.

Probability **In Exercises 68 and 69, the function**

$$f(x) = kx^n(1 - x)^m, \quad 0 \le x \le 1$$

where $n > 0$, $m > 0$, and k is a constant, can be used to represent various probability distributions. If k is chosen such that

$$\int_0^1 f(x)\,dx = 1$$

the probability that x will fall between a and b $(0 \le a \le b \le 1)$ is

$$P_{a,b} = \int_a^b f(x)\,dx.$$

68. The probability of recall in a certain experiment is

$$P_{a,b} = \int_a^b \frac{15}{4}x\sqrt{1 - x}\,dx$$

where x represents the percent of recall. (See figure.)

(a) For a randomly chosen individual, what is the probability that he or she will recall between 50% and 75% of the material?

(b) What is the median percent recall? That is, for what value of b is it true that the probability from 0 to b is 0.5?

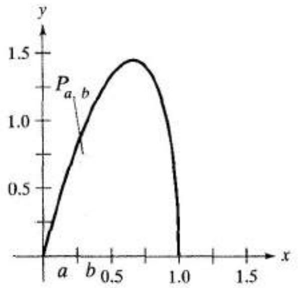


Figure for 68

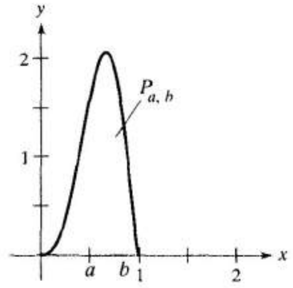


Figure for 69

69. The probability of finding iron in ore samples taken from a certain region is

$$P_{a,b} = \int_a^b \frac{1155}{32}x^3(1 - x)^{3/2}\,dx.$$

(See figure.) What is the probability that a sample will contain between

(a) 0% and 25%?

(b) 50% and 100%?

True or False? **In Exercises 70–74, determine whether the statement is true or false. If it is false, explain why or give an example that shows it is false.**

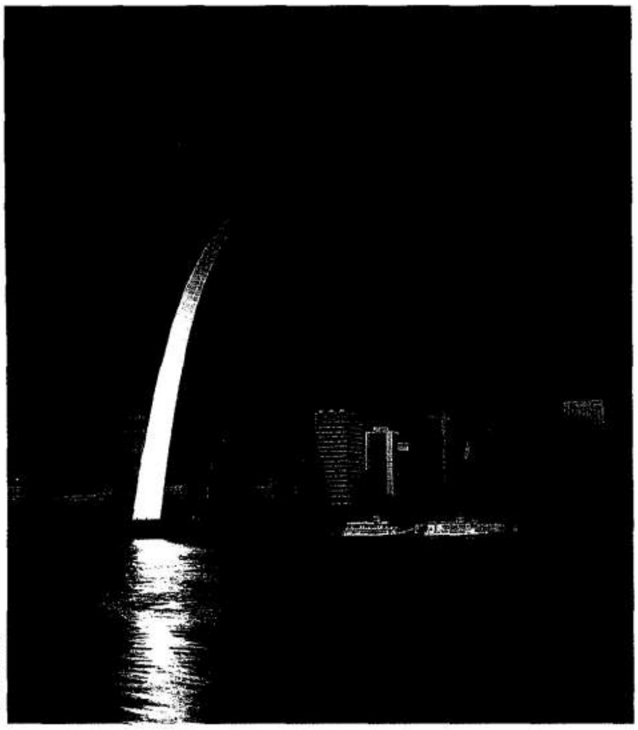
70. $\int x f(x)\,dx = x\int f(x)\,dx$

71. $\int (1/x)\,dx = -\left(\frac{1}{x^2}\right) + C$

72. If $f(x) = -f(-x)$ on the interval $[-a, a]$, then $\int_{-a}^{a} f(x)\,dx = 0$.

73. The average value of the sine function over an interval of length 2π is 0.

74. $\int \tan x\,dx = \sec^2 x + C$



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SECTION PROJECT

St. Louis Arch The Gateway Arch in St. Louis, Missouri was constructed using the hyperbolic cosine function. The equation used to construct the arch was

$$y = 693.8597 - 68.7672 \cosh 0.0100333x, \quad -299.2239 \le x \le 299.2239$$

where x and y are measured in feet. Cross sections of the arch are equilateral triangles and (x, y) traces the path of the centers of mass of the cross-sectional triangles. For each value of x, the area of the cross-sectional triangle is

$$A = 125.1406 \cosh 0.0100333x.$$

(*Source:* Owner's Manual for the Gateway Arch, *Saint Louis, MO, by William Thayer*)

(a) How high above the ground is the center of the highest triangle? (At ground level, $y = 0$.)

(b) What is the height of the arch? (*Hint:* For an equilateral triangle, $A = \sqrt{3}c^2$, where c is one-half the base of the triangle, and the center of mass of the triangle is located at two-thirds the height of the triangle.)

(c) How wide is the arch at ground level?

REVIEW EXERCISES FOR CHAPTER 5

In Exercises 1 and 2, sketch the graph of the function by hand. Identify any asymptotes of the graph.

1. $f(x) = \ln x + 3$
2. $f(x) = \ln(x - 3)$

In Exercises 3 and 4, use the properties of logarithms to write the expression as a sum, difference, and/or multiple of logarithms.

3. $\ln \sqrt[5]{\dfrac{4x^2 - 1}{4x^2 + 1}}$
4. $\ln[(x^2 + 1)(x - 1)]$

In Exercises 5 and 6, write the expression as the logarithm of a single quantity.

5. $\ln 3 + \frac{1}{3} \ln(4 - x^2) - \ln x$
6. $3[\ln x - 2 \ln(x^2 + 1)] + 2 \ln 5$

True or False? **In Exercises 7 and 8, determine whether the statement is true or false.**

7. The domain of the function $f(x) = \ln x$ is the set of all real numbers.
8. $\ln(x + y) = \ln x + \ln y$

In Exercises 9 and 10, solve the equation for x.

9. $\ln \sqrt{x + 1} = 2$
10. $\ln x + \ln(x - 3) = 0$

In Exercises 11–18, find the derivative of the function.

11. $g(x) = \ln \sqrt{x}$
12. $h(x) = \ln \dfrac{x(x - 1)}{x - 2}$
13. $f(x) = x\sqrt{\ln x}$
14. $f(x) = \ln[x(x^2 - 2)^{2/3}]$
15. $y = \dfrac{1}{b^2}\left[\ln(a + bx) + \dfrac{a}{a + bx}\right]$
16. $y = \dfrac{1}{b^2}[a + bx - a \ln(a + bx)]$
17. $y = -\dfrac{1}{a} \ln \dfrac{a + bx}{x}$
18. $y = -\dfrac{1}{ax} + \dfrac{b}{a^2} \ln \dfrac{a + bx}{x}$

In Exercises 19–26, evaluate the integral.

19. $\displaystyle\int \frac{1}{7x - 2}\, dx$
20. $\displaystyle\int \frac{x}{x^2 - 1}\, dx$
21. $\displaystyle\int \frac{\sin x}{1 + \cos x}\, dx$
22. $\displaystyle\int \frac{\ln \sqrt{x}}{x}\, dx$
23. $\displaystyle\int_1^4 \frac{x + 1}{x}\, dx$
24. $\displaystyle\int_1^e \frac{\ln x}{x}\, dx$
25. $\displaystyle\int_0^{\pi/3} \sec \theta \, d\theta$
26. $\displaystyle\int_0^{\pi/4} \tan\left(\frac{\pi}{4} - x\right) dx$

In Exercises 27–34, (a) find the inverse of the function, (b) use a graphing utility to graph f and f^{-1} in the same viewing rectangle, and (c) verify that $f^{-1}(f(x)) = f(f^{-1}(x)) = x$.

27. $f(x) = \frac{1}{2}x - 3$

28. $f(x) = 5x - 7$

29. $f(x) = \sqrt{x + 1}$

30. $f(x) = x^3 + 2$

31. $f(x) = \sqrt[3]{x + 1}$

32. $f(x) = x^2 - 5, \quad x \geq 0$

33. $f(x) = \ln \sqrt{x}$

34. $f(x) = e^{1-x}$

In Exercises 35–38, sketch the graph of the function by hand.

35. $y = e^{-x/2}$

36. $g(x) = 6(2^{-x^2})$

37. $h(x) = -3 \arcsin 2x$

38. $f(x) = 2 \arctan(x + 3)$

In Exercises 39 and 40, evaluate the expression without using a calculator. (*Hint:* Make a sketch of a right triangle.)

39. (a) $\sin\left(\arcsin \frac{1}{2}\right)$
(b) $\cos\left(\arcsin \frac{1}{2}\right)$

40. (a) $\tan(\operatorname{arccot} 2)$
(b) $\cos\left(\operatorname{arcsec} \sqrt{5}\right)$

In Exercises 41–58, find the derivative of the function.

41. $f(x) = \ln(e^{-x^2})$

42. $g(x) = \ln \dfrac{e^x}{1 + e^x}$

43. $g(t) = t^2 e^t$

44. $h(z) = e^{-z^2/2}$

45. $y = \sqrt{e^{2x} + e^{-2x}}$

46. $y = x^{2x+1}$

47. $f(x) = 3^{x-1}$

48. $f(x) = (4e)^x$

49. $g(x) = \dfrac{x^2}{e^x}$

50. $f(\theta) = \dfrac{1}{2} e^{\sin 2\theta}$

51. $y = \tan(\arcsin x)$

52. $y = \arctan(x^2 - 1)$

53. $y = x \operatorname{arcsec} x$

54. $y = \frac{1}{2} \arctan e^{2x}$

55. $y = x(\arcsin x)^2 - 2x + 2\sqrt{1 - x^2} \arcsin x$

56. $y = \sqrt{x^2 - 4} - 2 \operatorname{arcsec}(x/2), \quad 2 < x < 4$

57. $y = 2x - \cosh \sqrt{x}$

58. $y = x \tanh^{-1} 2x$

In Exercises 59 and 60, use implicit differentiation to find dy/dx.

59. $y \ln x + y^2 = 0$

60. $\cos x^2 = xe^y$

61. ***Think About It*** Find the derivative of each function, given that a is constant.

(a) $y = x^a$, (b) $y = a^x$, (c) $y = x^x$, (d) $y = a^a$

62. ***Compound Interest*** How large a deposit, at 7 percent interest compounded continuously, must be made to obtain a balance of \$10,000 in 15 years?

63. ***Compound Interest*** A deposit earns interest at a rate of r percent compounded continuously and doubles in value in 10 years. Find r.

64. ***Climb Rate*** The time t (in minutes) for a small plane to climb to an altitude of h feet is

$$t = 50 \log_{10} \frac{18{,}000}{18{,}000 - h}$$

where 18,000 feet is the plane's absolute ceiling.

(a) Determine the domain of the function appropriate for the context of the problem.

(b) Use a graphing utility to graph the time function and identify any asymptotes.

(c) As the plane approaches its absolute ceiling, what can be concluded about the time required to further increase its altitude?

(d) Find the time when the altitude is increasing at the greatest rate.

In Exercises 65–80, evaluate the integral.

65. $\displaystyle\int xe^{-3x^2}\,dx$

66. $\displaystyle\int \frac{e^{1/x}}{x^2}\,dx$

67. $\displaystyle\int \frac{e^{4x} - e^{2x} + 1}{e^x}\,dx$

68. $\displaystyle\int \frac{e^{2x} - e^{-2x}}{e^{2x} + e^{-2x}}\,dx$

69. $\displaystyle\int \frac{e^x}{e^x - 1}\,dx$

70. $\displaystyle\int x^2 e^{x^3+1}\,dx$

71. $\displaystyle\int \frac{1}{e^{2x} + e^{-2x}}\,dx$

72. $\displaystyle\int \frac{1}{3 + 25x^2}\,dx$

73. $\displaystyle\int \frac{x}{\sqrt{1 - x^4}}\,dx$

74. $\displaystyle\int \frac{1}{16 + x^2}\,dx$

75. $\displaystyle\int \frac{x}{16 + x^2}\,dx$

76. $\displaystyle\int \frac{4 - x}{\sqrt{4 - x^2}}\,dx$

77. $\displaystyle\int \frac{\arctan(x/2)}{4 + x^2}\,dx$

78. $\displaystyle\int \frac{\arcsin x}{\sqrt{1 - x^2}}\,dx$

79. $\displaystyle\int \frac{x}{\sqrt{x^4 - 1}}\,dx$

80. $\displaystyle\int x^2 \operatorname{sech}^2 x^3\,dx$

In Exercises 81 and 82, find the area of the region bounded by the graphs of the equations.

81. $y = xe^{-x^2}, \quad y = 0, \quad x = 0, \quad x = 4$

82. $y = \dfrac{1}{x^2 + 1}, \quad y = 0, \quad x = 0, \quad x = 1$

In Exercises 83–88, solve the differential equation.

83. $\dfrac{dy}{dx} = \dfrac{x^2 + 3}{x}$

84. $\dfrac{dy}{dx} = \dfrac{e^{-2x}}{1 + e^{-2x}}$

85. $y' - 2xy = 0$

86. $y' - e^y \sin x = 0$

87. $\dfrac{dy}{dx} = \dfrac{x^2 + y^2}{2xy}$

88. $\dfrac{dy}{dx} = \dfrac{3(x + y)}{x}$