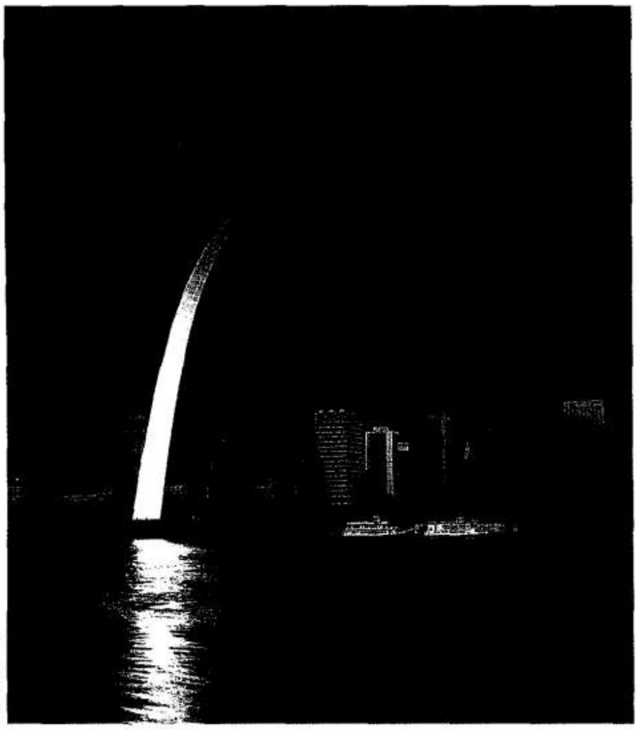

SECTION PROJECT

St. Louis Arch The Gateway Arch in St. Louis, Missouri was constructed using the hyperbolic cosine function. The equation used to construct the arch was

$$y = 693.8597 - 68.7672 \cosh 0.0100333x,$$
$$-299.2239 \le x \le 299.2239$$

where x and y are measured in feet. Cross sections of the arch are equilateral triangles and (x, y) traces the path of the centers of mass of the cross-sectional triangles. For each value of x, the area of the cross-sectional triangle is

$$A = 125.1406 \cosh 0.0100333x.$$

(Source: Owner's Manual for the Gateway Arch, *Saint Louis, MO, by William Thayer)*

(a) How high above the ground is the center of the highest triangle? (At ground level, $y = 0$.)

(b) What is the height of the arch? (*Hint:* For an equilateral triangle, $A = \sqrt{3}c^2$, where c is one-half the base of the triangle, and the center of mass of the triangle is located at two-thirds the height of the triangle.)

(c) How wide is the arch at ground level?

REVIEW EXERCISES FOR CHAPTER 5

In Exercises 1 and 2, sketch the graph of the function by hand. Identify any asymptotes of the graph.

1. $f(x) = \ln x + 3$

2. $f(x) = \ln(x - 3)$

In Exercises 3 and 4, use the properties of logarithms to write the expression as a sum, difference, and/or multiple of logarithms.

3. $\ln \sqrt[5]{\dfrac{4x^2 - 1}{4x^2 + 1}}$

4. $\ln[(x^2 + 1)(x - 1)]$

In Exercises 5 and 6, write the expression as the logarithm of a single quantity.

5. $\ln 3 + \frac{1}{3}\ln(4 - x^2) - \ln x$

6. $3[\ln x - 2\ln(x^2 + 1)] + 2\ln 5$

True or False? **In Exercises 7 and 8, determine whether the statement is true or false.**

7. The domain of the function $f(x) = \ln x$ is the set of all real numbers.

8. $\ln(x + y) = \ln x + \ln y$

In Exercises 9 and 10, solve the equation for x.

9. $\ln \sqrt{x + 1} = 2$

10. $\ln x + \ln(x - 3) = 0$

In Exercises 11–18, find the derivative of the function.

11. $g(x) = \ln \sqrt{x}$

12. $h(x) = \ln \dfrac{x(x - 1)}{x - 2}$

13. $f(x) = x\sqrt{\ln x}$

14. $f(x) = \ln[x(x^2 - 2)^{2/3}]$

15. $y = \dfrac{1}{b^2}\left[\ln(a + bx) + \dfrac{a}{a + bx}\right]$

16. $y = \dfrac{1}{b^2}[a + bx - a\ln(a + bx)]$

17. $y = -\dfrac{1}{a}\ln \dfrac{a + bx}{x}$

18. $y = -\dfrac{1}{ax} + \dfrac{b}{a^2}\ln \dfrac{a + bx}{x}$

In Exercises 19–26, evaluate the integral.

19. $\displaystyle\int \frac{1}{7x - 2}\,dx$

20. $\displaystyle\int \frac{x}{x^2 - 1}\,dx$

21. $\displaystyle\int \frac{\sin x}{1 + \cos x}\,dx$

22. $\displaystyle\int \frac{\ln \sqrt{x}}{x}\,dx$

23. $\displaystyle\int_1^4 \frac{x + 1}{x}\,dx$

24. $\displaystyle\int_1^e \frac{\ln x}{x}\,dx$

25. $\displaystyle\int_0^{\pi/3} \sec \theta \, d\theta$

26. $\displaystyle\int_0^{\pi/4} \tan\left(\frac{\pi}{4} - x\right) dx$

In Exercises 27–34, (a) find the inverse of the function, (b) use a graphing utility to graph f and f^{-1} in the same viewing rectangle, and (c) verify that $f^{-1}(f(x)) = f(f^{-1}(x)) = x$.

27. $f(x) = \frac{1}{2}x - 3$
28. $f(x) = 5x - 7$
29. $f(x) = \sqrt{x + 1}$
30. $f(x) = x^3 + 2$
31. $f(x) = \sqrt[3]{x + 1}$
32. $f(x) = x^2 - 5, \quad x \geq 0$
33. $f(x) = \ln \sqrt{x}$
34. $f(x) = e^{1-x}$

In Exercises 35–38, sketch the graph of the function by hand.

35. $y = e^{-x/2}$
36. $g(x) = 6(2^{-x^2})$
37. $h(x) = -3 \arcsin 2x$
38. $f(x) = 2 \arctan(x + 3)$

In Exercises 39 and 40, evaluate the expression without using a calculator. (*Hint:* Make a sketch of a right triangle.)

39. (a) $\sin\left(\arcsin \frac{1}{2}\right)$
    (b) $\cos\left(\arcsin \frac{1}{2}\right)$
40. (a) $\tan(\operatorname{arccot} 2)$
    (b) $\cos\left(\operatorname{arcsec} \sqrt{5}\right)$

In Exercises 41–58, find the derivative of the function.

41. $f(x) = \ln(e^{-x^2})$
42. $g(x) = \ln \dfrac{e^x}{1 + e^x}$
43. $g(t) = t^2 e^t$
44. $h(z) = e^{-z^2/2}$
45. $y = \sqrt{e^{2x} + e^{-2x}}$
46. $y = x^{2x+1}$
47. $f(x) = 3^{x-1}$
48. $f(x) = (4e)^x$
49. $g(x) = \dfrac{x^2}{e^x}$
50. $f(\theta) = \dfrac{1}{2}e^{\sin 2\theta}$
51. $y = \tan(\arcsin x)$
52. $y = \arctan(x^2 - 1)$
53. $y = x \operatorname{arcsec} x$
54. $y = \frac{1}{2} \arctan e^{2x}$
55. $y = x(\arcsin x)^2 - 2x + 2\sqrt{1 - x^2} \arcsin x$
56. $y = \sqrt{x^2 - 4} - 2 \operatorname{arcsec}(x/2), \quad 2 < x < 4$
57. $y = 2x - \cosh \sqrt{x}$
58. $y = x \tanh^{-1} 2x$

In Exercises 59 and 60, use implicit differentiation to find dy/dx.

59. $y \ln x + y^2 = 0$
60. $\cos x^2 = xe^y$

61. ***Think About It*** Find the derivative of each function, given that a is constant.
    (a) $y = x^a$, (b) $y = a^x$, (c) $y = x^x$, (d) $y = a^a$

62. ***Compound Interest*** How large a deposit, at 7 percent interest compounded continuously, must be made to obtain a balance of \$10,000 in 15 years?

63. ***Compound Interest*** A deposit earns interest at a rate of r percent compounded continuously and doubles in value in 10 years. Find r.

64. ***Climb Rate*** The time t (in minutes) for a small plane to climb to an altitude of h feet is

$$t = 50 \log_{10} \frac{18{,}000}{18{,}000 - h}$$

where 18,000 feet is the plane's absolute ceiling.

(a) Determine the domain of the function appropriate for the context of the problem.
(b) Use a graphing utility to graph the time function and identify any asymptotes.
(c) As the plane approaches its absolute ceiling, what can be concluded about the time required to further increase its altitude?
(d) Find the time when the altitude is increasing at the greatest rate.

In Exercises 65–80, evaluate the integral.

65. $\displaystyle\int xe^{-3x^2}\, dx$
66. $\displaystyle\int \frac{e^{1/x}}{x^2}\, dx$
67. $\displaystyle\int \frac{e^{4x} - e^{2x} + 1}{e^x}\, dx$
68. $\displaystyle\int \frac{e^{2x} - e^{-2x}}{e^{2x} + e^{-2x}}\, dx$
69. $\displaystyle\int \frac{e^x}{e^x - 1}\, dx$
70. $\displaystyle\int x^2 e^{x^3+1}\, dx$
71. $\displaystyle\int \frac{1}{e^{2x} + e^{-2x}}\, dx$
72. $\displaystyle\int \frac{1}{3 + 25x^2}\, dx$
73. $\displaystyle\int \frac{x}{\sqrt{1 - x^4}}\, dx$
74. $\displaystyle\int \frac{1}{16 + x^2}\, dx$
75. $\displaystyle\int \frac{x}{16 + x^2}\, dx$
76. $\displaystyle\int \frac{4 - x}{\sqrt{4 - x^2}}\, dx$
77. $\displaystyle\int \frac{\arctan(x/2)}{4 + x^2}\, dx$
78. $\displaystyle\int \frac{\arcsin x}{\sqrt{1 - x^2}}\, dx$
79. $\displaystyle\int \frac{x}{\sqrt{x^4 - 1}}\, dx$
80. $\displaystyle\int x^2 \operatorname{sech}^2 x^3\, dx$

In Exercises 81 and 82, find the area of the region bounded by the graphs of the equations.

81. $y = xe^{-x^2}, \quad y = 0, \quad x = 0, \quad x = 4$
82. $y = \dfrac{1}{x^2 + 1}, \quad y = 0, \quad x = 0, \quad x = 1$

In Exercises 83–88, solve the differential equation.

83. $\dfrac{dy}{dx} = \dfrac{x^2 + 3}{x}$
84. $\dfrac{dy}{dx} = \dfrac{e^{-2x}}{1 + e^{-2x}}$
85. $y' - 2xy = 0$
86. $y' - e^y \sin x = 0$
87. $\dfrac{dy}{dx} = \dfrac{x^2 + y^2}{2xy}$
88. $\dfrac{dy}{dx} = \dfrac{3(x + y)}{x}$