

AP Practice Using Integration for Accumulation

Question 1

- (a) Solve $|v(t)| = 2$ on $2 \leq t \leq 4$.
 $t = 3.128$ (or 3.127) and $t = 3.473$

2 : $\begin{cases} 1 : \text{considers } |v(t)| = 2 \\ 1 : \text{answer} \end{cases}$

- (b) $s(t) = 10 + \int_0^t v(x) dx$

2 : $\begin{cases} 1 : s(t) \\ 1 : s(5) \end{cases}$

$$s(5) = 10 + \int_0^5 v(x) dx = -9.207$$

- (c) $v(t) = 0$ when $t = 0.536033, 3.317756$
 $v(t)$ changes sign from negative to positive at time $t = 0.536033$.
 $v(t)$ changes sign from positive to negative at time $t = 3.317756$.

3 : $\begin{cases} 1 : \text{considers } v(t) = 0 \\ 2 : \text{answers with justification} \end{cases}$

Therefore, the particle changes direction at time $t = 0.536$ and time $t = 3.318$ (or 3.317).

- (d) $v(4) = -11.475758 < 0$, $a(4) = v'(4) = -22.295714 < 0$

2 : conclusion with reason

The speed is increasing at time $t = 4$ because velocity and acceleration have the same sign.

Question 2

(a) $\int_0^{12} P(t) dt \approx 46 \cdot 4 + 57 \cdot 4 + 62 \cdot 4 = 660 \text{ ft}^3$

2 : $\begin{cases} 1 : \text{midpoint sum} \\ 1 : \text{answer} \end{cases}$

(b) $\int_0^{12} R(t) dt = 225.594 \text{ ft}^3$

2 : $\begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

(c) $1000 + \int_0^{12} P(t) dt - \int_0^{12} R(t) dt = 1434.406$

1 : answer

At time $t = 12$ hours, the volume of water in the pool is approximately 1434 ft^3 .

Question 3

(a) The squirrel changes direction whenever its velocity changes sign. This occurs at $t = 9$ and $t = 15$.

2 : $\begin{cases} 1 : t\text{-values} \\ 1 : \text{explanation} \end{cases}$

(b) Velocity is 0 at $t = 0$, $t = 9$, and $t = 15$.

2 : $\begin{cases} 1 : \text{identifies candidates} \\ 1 : \text{answers} \end{cases}$

t	position at time t
0	0
9	$\frac{9+5}{2} \cdot 20 = 140$
15	$140 - \frac{6+4}{2} \cdot 10 = 90$
18	$90 + \frac{3+2}{2} \cdot 10 = 115$

The squirrel is farthest from building A at time $t = 9$; its greatest distance from the building is 140.

(c) The total distance traveled is $\int_0^{18} |v(t)| dt = 140 + 50 + 25 = 215$.

1 : answer