

## Volume of Known Cross Sections

W-up: Find the distance between  $f(x)=\sqrt{x}$  and  $g(x)=\frac{1}{4}x^2$  when  $x$  equals:

- A) 0                      B) 1                      C) 2                      D) *at any x*

Find the distance between  $f(x)=\sqrt{x}$  and  $g(x)=\frac{1}{4}x^2$  when  $y$  equals:

- A) 0                      B) .5                      C) 1                      D) *at any y*

**Volume of Known Cross Sections** : Finding the volume of a solid created by adding an infinite number of similar figures of the same type (these are the known cross sections).

### Volume of a cross sections perpendicular to the $x$ -axis

$$V = \int_a^b A(x) dx$$

Where  $A(x)$  represents the algebraic expression for the area of any cross section on  $[a, b]$

### Volume of a cross sections perpendicular to the $y$ -axis

$$V = \int_c^d A(y) dy$$

Where  $A(y)$  represents the algebraic expression for the area of any cross section on  $[c, d]$

### ***Commonly used Area Formulas***

Square:  $A = b^2$

Circle:  $A = \pi r^2$

Rectangle:  $A = b \cdot h$

Trapezoid:  $A = \frac{1}{2}(b_1 + b_2) \cdot h$

Triangle:  $A = \frac{1}{2}b \cdot h$

Ellipse:  $A = \pi \cdot b \cdot h$

Equilateral Triangle:  $A = \frac{\sqrt{3}}{4}b^2$

Isosceles Right Triangle:  $A = \frac{1}{4}b^2$

Note: “ $b$ ” represents the functional expression for distance between curves!

**EX)** The region bounded by the curves  $f(x) = \sqrt{x}$  and  $g(x) = \frac{1}{4}x^2$  is the base of a solid.

**A)** Find the volume of a solid whose cross sections perpendicular to the  $x$ -axis are:

- 1) squares
- 2) rectangles of height 3
- 3) semicircles

**B)** Find the volume of a solid whose cross sections perpendicular to the  $y$ -axis are:

- 1) triangles of height 5
- 2) equilateral triangles