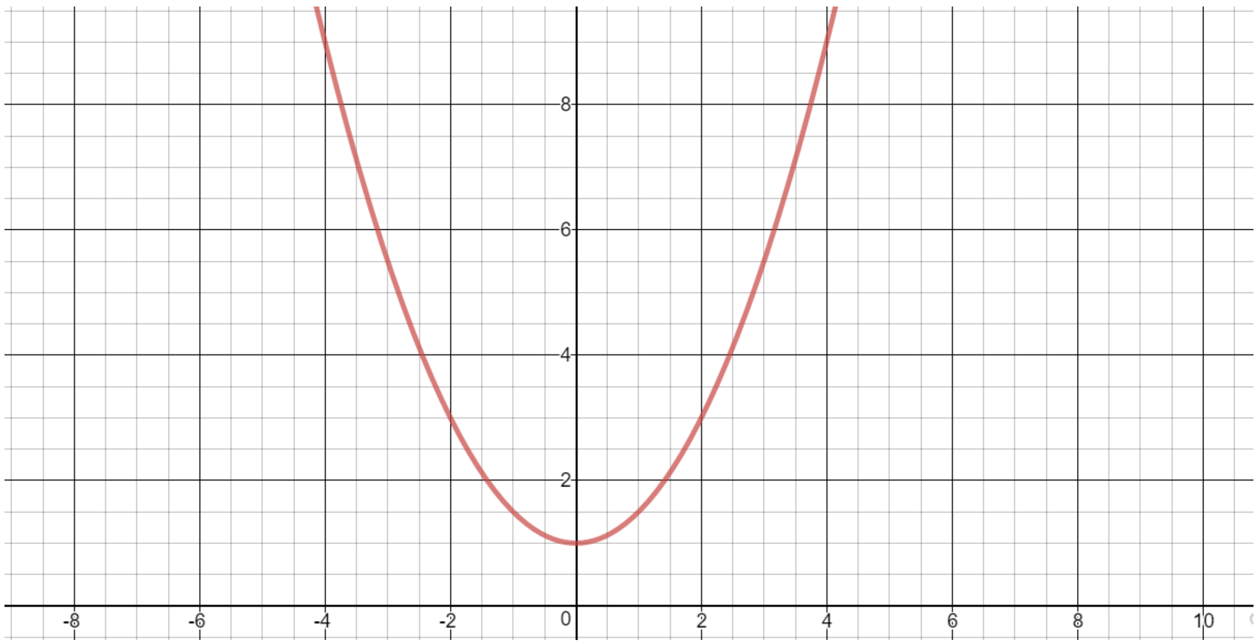
Volume of Rotational Solids

W-up: Use the graph of the function $f(x)=\frac{1}{2}x^2+1$ to answer the following questions.



1) Use this graph to find the distance between $f(x)$ and the x-axis when x equals:

A) 1   B) 2   C) -2   D) ***at any x***

Lastly, use what you have learned to give an expression for the distance between $f(x)$ and the line y = 1 ***at any x.***

2) Use this graph to find the 1st quadrant distance between $f(x)$ and the y-axis when y equals:

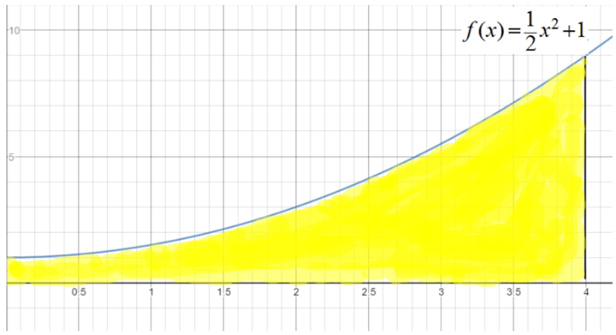
A) 1   B) 5.5   C) 9   D) ***at any y***

Lastly, use what you have learned to give an expression for the distance between $f(x)$ and the line x = 5 ***at any y.***

Disc Method for Solids

The disc method of finding volumes of rotational solids is used **when the axis of rotation is ALSO a border of the shaded region**(cross sections are discs).

EX) Find the volume of a solid created by revolving the region bounded by the graphs of $f(x) = \frac{1}{2}x^2 + 1$, $x = 0$, $x = 4$ and the x-axis about the x-axis.



The height of each function represents a singular radius of a circle. Since the expression $\frac{1}{2}x^2 + 1$ represents that radius, $\pi \bullet \left(\frac{1}{2}x^2 + 1\right)^2$ is an expression for the area of that circle. Secondly, because volume is the accumulation of area,

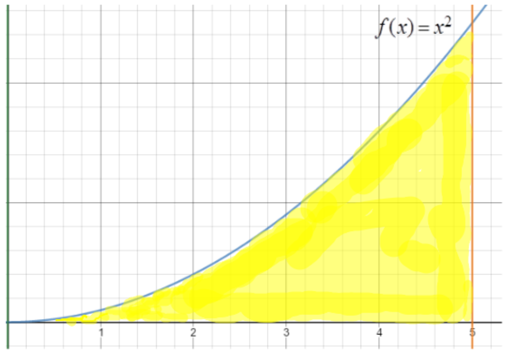
V= $\int_0^4 \pi \bullet \left(\frac{1}{2}x^2 + 1\right)^2 \, dx$ as every x-value from 0 to 4 is used for x and added together!

Volume of a rotational solid using discs

$$V = \pi \int_a^b \left[\mathrm{R}(x) \right]^2 \, dx$$

Where $\mathrm{R}(x)$ represents the algebraic expression for the radius of any circle created by the rotation on [a, b]

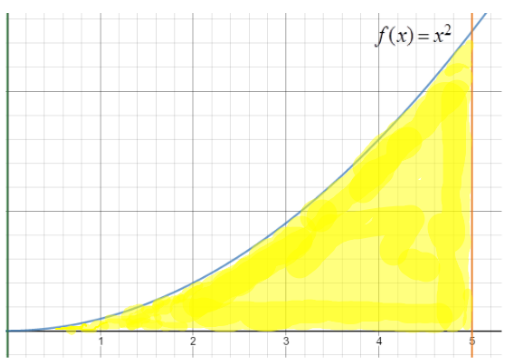
EX) Find the volume of a solid created by revolving the region bounded by the graphs of $f(x) = x^2$, $x = 0$, $x = 5$ and the x-axis about the x-axis.



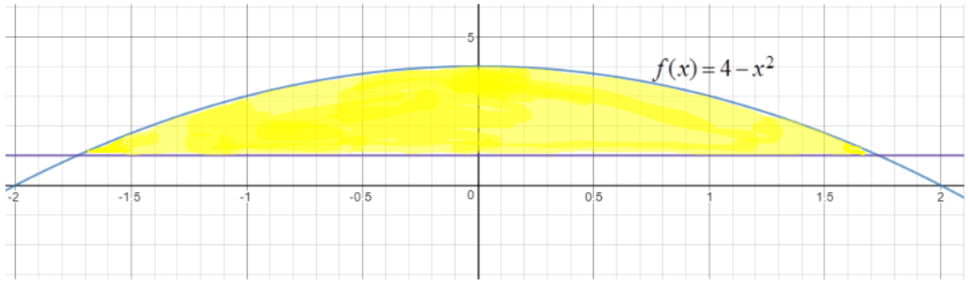
Note: when revolving about the ***y*-axis(or other vertical line)** the volume formula remains the same except ALL values and expressions must be in terms of y!

$$V = \pi \int_c^d \left[\mathrm{R(y)} \right]^2 dy$$

EX) Find the volume of a solid created by revolving the region bounded by the graphs of $f(x) = x^2$, $x = 0$, $x = 5$ and the x-axis about the line $x = 5$.



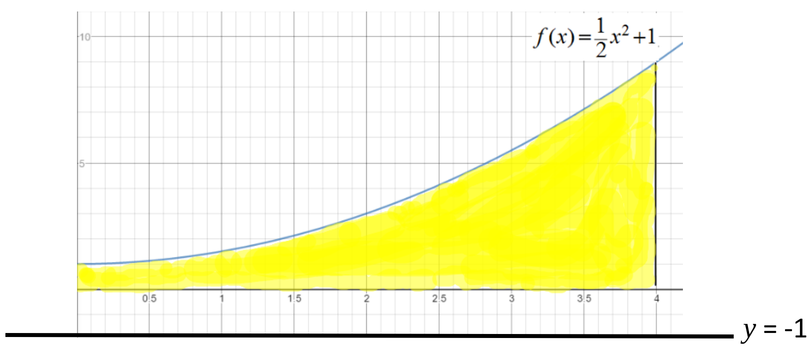
EX) Find the volume of a solid created by revolving the region bounded by the graphs of $f(x) = 4 - x^2$, $y = 1$, *about* the line $y = 1$.



Washer Method for Solids

The washer method of finding volumes of rotational solids is used **when the the axis of rotation IS NOT the border of the shaded region**(cross sections are washers)

EX) Find the volume of a solid created by revolving the region bounded by the graphs of $f(x) = \frac{1}{2}x^2 + 1$, $x = 0$, $x = 4$ and the x-axis **about** the line $y = -1$



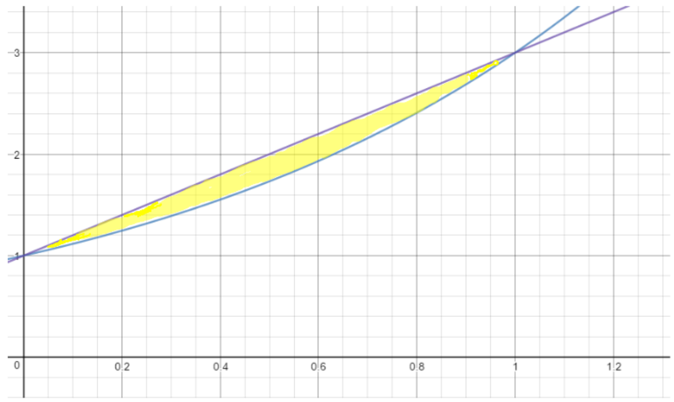
Volume of a rotational solid using washers

$$V = \pi \int_a^b \left[\mathrm{R}(x) \right]^2 - \left[\mathrm{r}(x) \right]^2 dx$$

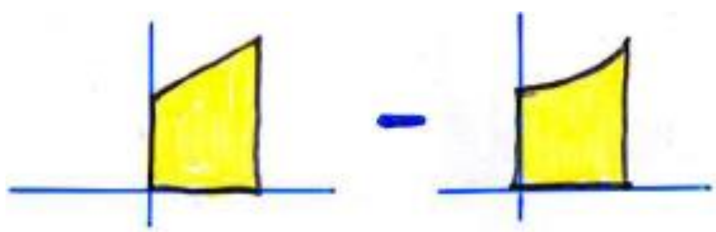
Where $\mathrm{R}(x)$ represents the algebraic expression for the radius of the "OUTER" curve to the axis of rotation

and $\mathrm{r}(x)$ represents the algebraic expression for the radius of the "INNER" curve to the axis of rotation.

EX) Find the volume of a solid created by revolving the region bounded by the graphs of $f(x)=2x+1$ and $g(x)=3^x$ about the x-axis.



Use the ***difference*** of the TWO solids created from rotating each function about the axis of rotation. Be sure the larger solid(radius created from the outer function) is first!



Volume of a solid that is too large - Volume of the hole

$$V = \pi\int_0^1 \left(2x+1\right)^2 dx - \pi\int_0^1 \left(3^x\right)^2 dx$$

Or

$$V = \pi\int_0^1 \left(2x+1\right)^2 - \left(3^{2x}\right) dx$$

EX) Find the volume of a solid created by revolving the region bounded by the graphs of $f(x)=3\sqrt{x}$ and $g(x)=x$ about:

A) the x-axis

B) the y-axis

C) the line y = 9

D) the line y = 12

E) the line x = 12

F) the line x = -2