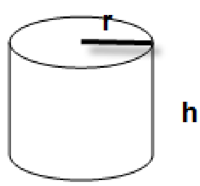
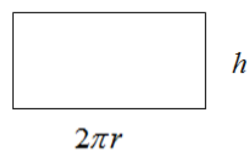
Using the Shell Method for Finding Rotational Volumes

The Shell Method: Used when it is convenient to represent the radius ***in terms of x*** but rotation is about a vertical axis(like the y-axis) or vice-versa. Also NEEDED when an equation cannot be solved **for a specific variable**.

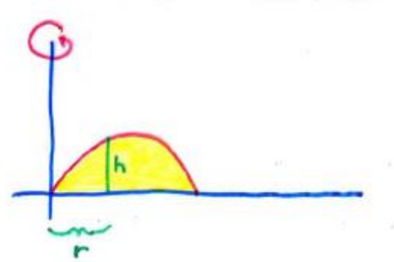
EX) Find the volume of a solid of revolution formed by revolving the 1st quadrant region bounded by $f(x)=x-x^3$ and $y=0$ about the y-axis (to be solved later).

The shell method uses the lateral area of cylinders to find volumes





Lateral Area = $2\pi rh$



Shell Method for **Vertical** Axis of Revolution

$$V=2\pi\int_a^b p(x)\bullet h(x)\,dx$$

Where $p(x)$ represents the algebraic expression for the radius of each cylinder(distance between the *axis of revolution* and *any "x"*) on [a, b] AND $h(x)$ represents the algebraic expression for the height of each cylinder (distance between boundaries of the bounded region) on [a, b].

Shell Method for **Horizontal** Axis of Revolution

$$V = 2\pi \int_c^d p(y) \bullet h(y)\, dy$$

Where $p(y)$ represents the algebraic expression for the radius of each cylinder(distance between the *axis of revolution* and *any "y"*) on $[c, d]$ AND $h(y)$ represents the algebraic expression for the height of each cylinder (distance between boundaries of the bounded region) on $[c, d]$.

Remember: When finding vertical distance(which is always positive) on the coordinate plane use the "top value – bottom value" and when finding horizontal distance use the "right value – left value."

EX) Find the volume of a solid of revolution formed by revolving the 1st quadrant region bounded by $f(x) = x - x^3$ and $y = 0$ about the y-axis

Now, same region above rotated about the line $x = -1$.

Now, same region above rotated about the line $x = 2$.

EX) Find the volume of a solid of revolution formed by revolving the 1st quadrant region bounded by $f(x) = 2x^2 + 3$, $x = 0$ and $y = 7$ about the y-axis

Now, same region above rotated about the line $x = -4$.

Now, same region above rotated about the line $x = 4$.