Unit 4 AP Classroom Practice for Lessons 1-5

1) Which of the following is the midpoint Riemann sum approximation of $\int_4^6 \sqrt{x^3+1}\,dx$ using 4 subintervals of equal width?

(A) $\frac{1}{4}\left(\sqrt{4.25^3+1}+\sqrt{4.75^3+1}+\sqrt{5.25^3+1}+\sqrt{5.75^3+1}\right)$

(B) $\frac{1}{2}\left(\sqrt{4.25^3+1}+\sqrt{4.75^3+1}+\sqrt{5.25^3+1}+\sqrt{5.75^3+1}\right)$

(C) $\frac{1}{4}\left(\frac{\sqrt{4^3+1}+\sqrt{4.5^3+1}}{2}+\frac{\sqrt{4.5^3+1}+\sqrt{5^3+1}}{2}+\frac{\sqrt{5^3+1}+\sqrt{5.5^3+1}}{2}+\frac{\sqrt{5.5^3+1}+\sqrt{6^3+1}}{2}\right)$

(D) $\frac{1}{2}\left(\frac{\sqrt{4^3+1}+\sqrt{4.5^3+1}}{2}+\frac{\sqrt{4.5^3+1}+\sqrt{5^3+1}}{2}+\frac{\sqrt{5^3+1}+\sqrt{5.5^3+1}}{2}+\frac{\sqrt{5.5^3+1}+\sqrt{6^3+1}}{2}\right)$

2)

t	1	5	8	13
$R(t)$	1	10	17	32

The differentiable function R is increasing, and the graph of R is concave down. Values of $R(t)$ at selected values of t are given in the table above. The trapezoidal sum $(5-1)\left(\frac{10+1}{2}\right)+(8-5)\left(\frac{17+10}{2}\right)+(13-8)\left(\frac{17+32}{2}\right)$ approximates $\int_1^{13} R(t)\,dt$. Which of the following statements is true?

(A) The trapezoidal sum is an underestimate for $\int_1^{13} R(t)\,dt$ because R is increasing.

(B) The trapezoidal sum is an overestimate for $\int_1^{13} R(t)\,dt$ because R is increasing.

(C) The trapezoidal sum is an underestimate for $\int_1^{13} R(t)\,dt$ because the graph of R is concave down.

(D) The trapezoidal sum is an overestimate for $\int_1^{13} R(t)\,dt$ because the graph of R is concave down.

3) **Approximate the area between the x-axis and $f(x)$ from $x=-2$ to $x=9$ using a *left Riemann sum* with 4 unequal subdivisions.**

x	-2	0	2	6	9
$f(x)$	7	11	2	8	1

The approximate area is ______ units^2.

4) Which of the following definite integrals is equal to $\lim_{n\to\infty}\sum_{k=1}^{n}\frac{10k}{n}\left(\sqrt{1+\frac{5k}{n}}\right)\left(\frac{5}{n}\right)$?

Ⓐ $\int_1^6 10\sqrt{x}dx$

Ⓑ $\int_1^6 2x\sqrt{x}dx$

Ⓒ $\int_0^5 10\sqrt{1+x}dx$

Ⓓ $\int_0^5 2x\sqrt{1+x}dx$

5) Which of the following is a left Riemann sum approximation of $\int_2^8 \cos\left(x^2\right)dx$ with n subintervals of equal length? (Careful, this asks for LEFT sum)

Ⓐ $\sum_{k=1}^{n}\left(\cos\left(2+\frac{k-1}{n}\right)^2\right)\frac{1}{n}$

Ⓑ $\sum_{k=1}^{n}\left(\cos\left(\frac{6k}{n}\right)^2\right)\frac{6}{n}$

Ⓒ $\sum_{k=1}^{n}\left(\cos\left(2+\frac{6(k-1)}{n}\right)^2\right)\frac{6}{n}$

Ⓓ $\sum_{k=1}^{n}\left(\cos\left(2+\frac{6k}{n}\right)^2\right)\frac{6}{n}$

6) Which of the following definite integrals are equal to $\lim_{n\to\infty}\sum_{k=1}^{n}\left(-2+\frac{8k}{n}\right)^3\frac{8}{n}$?

I. $\int_{-2}^{6} x^3 dx$

II. $\int_0^8 (-2+x)^3 dx$

III. $\int_0^1 8(-2+8x)^3 dx$

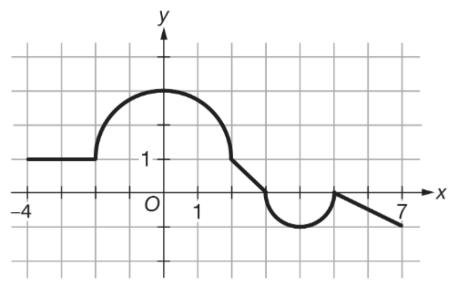
Ⓐ I only

Ⓑ II only

Ⓒ III only

Ⓓ I, II, and III

7)



Graph of f

The graph of the function f on the interval $-4 \le x \le 7$ consists of three line segments and two semicircles, as shown in the figure above. What is the value of $\int_{-4}^{7} f(x)\,dx$?

Ⓐ $\frac{3}{2}\pi + \frac{3}{2}$

Ⓑ $\frac{3}{2}\pi + \frac{11}{2}$

Ⓒ $\frac{5}{2}\pi + \frac{7}{2}$

Ⓓ $\frac{5}{2}\pi + \frac{15}{2}$

8) If $\int_{-2}^{8} (3g(x) + 2)\,dx = 35$ and $\int_{5}^{-2} g(x)\,dx = -12$, then $\int_{5}^{8} g(x)\,dx =$

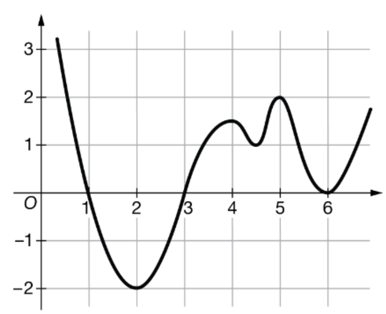
Ⓐ -23

Ⓑ -7

Ⓒ 19

Ⓓ 23

9)



Graph of f

The graph of the function f is shown above. Let g be the function defined by $g(x) = \int_{1}^{x} f(t)\,dt$. At what values of x in the interval $0.5 < x < 6.5$ does g have a relative maximum?

Ⓐ 1 only

Ⓑ 3 only

Ⓒ 4 and 5

Ⓓ 1, 3, and 6

10) Let f be the function given by $f(x) = \int_6^x (-t^2 - t + 6)\,dt$. On which of the following intervals is f increasing?

Ⓐ $(-\infty, -\frac{1}{2}]$

Ⓑ $[-3, 2]$

Ⓒ $[-\frac{1}{2}, \infty)$

Ⓓ $(-\infty, -3]$ and $[2, \infty)$

11) If $h(x) = \int_{-1}^{x^3} \sqrt{2 + t^2}\,dt$ for $x \geq 0$, then $h'(x) =$

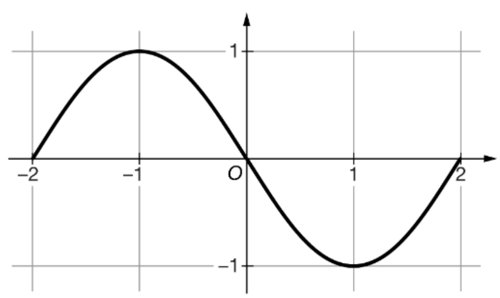
Ⓐ $\sqrt{2 + x^2}$

Ⓑ $\sqrt{2 + x^6}$

Ⓒ $3x^2\sqrt{2 + x^2}$

Ⓓ $3x^2\sqrt{2 + x^6}$

12)



Graph of f

The graph of the function f defined on the closed interval $[-2, 2]$ is shown above. Let g be defined by $g(x) = \int_0^x f(t)\,dt$. On which of the following intervals is the graph of g both decreasing and concave up?

Ⓐ $(-2, -1)$

Ⓑ $(-1, 0)$

Ⓒ $(0, 1)$

Ⓓ $(1, 2)$

13)

$$f(x) = \begin{cases} 2 & \text{for } 0 \le x < 2 \\ 3 & \text{for } 2 < x < 3 \\ -1 & \text{for } 3 \le x \le 8 \end{cases}$$

Let f be the piecewise function given above. The value of $\int_0^8 f(x)dx$ is

Ⓐ 2

Ⓑ 12

Ⓒ 32

Ⓓ nonexistent

14)

x	2	6	7	9	11	13
$h(x)$	16	-8	-9	-5	7	27
$h'(x)$	-10	-2	0	4	8	12

Selected values of the differentiable function h and its first derivative h' are given in the table above. Let f be the function defined by $f(x) = \int_0^x h(t)dt$. What is the least possible number of critical points for f in the interval $2 \le x \le 13$?

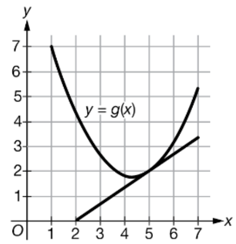
Ⓐ Zero

Ⓑ One

Ⓒ Two

Ⓓ It cannot be determined from the information given.

15)



The figure above shows the graph of the differentiable function g and the line tangent to the graph of g at the point $(5, 2)$. Let f be the function given by $f(x) = \int_0^x g(t)dt$. Let h be the function with first derivative given by $h'(x) = x^2 \sin\left(\frac{x}{3}\right)$ for $0 < x < 7$. If the line tangent to the graph of h at $x = a$ is parallel to the line tangent to the graph of f at $x = 5$, what is the value of a ?

Ⓐ 1.273

Ⓑ 1.857

Ⓒ 2.572

Ⓓ 24.885

16)

x	0	2	4	6
$f(x)$	-22	-6	2	2
$f'(x)$	10	6	2	-2

Selected values of the twice-differentiable function f and its derivative f' are given in the table above. What is the value of $\int_0^6 f'(x)dx$?

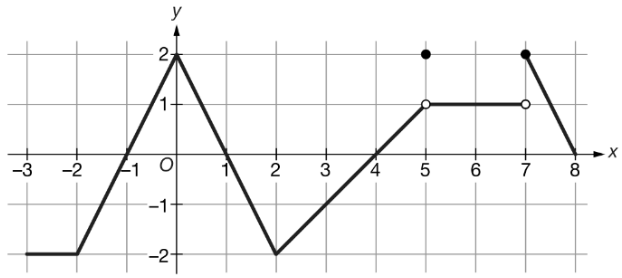
Ⓐ -12

Ⓑ 12

Ⓒ 24

Ⓓ 36

17) **OPEN ENDED (Calculator NOT Allowed)**



Graph of f

The graph of the function f on the closed interval $-3 \leq x \leq 8$ consists of six line segments and the point $(5, 2)$, as shown in the figure above. The function g is given by $g(x) = \frac{1}{10}(4x^3 + 3x^2 - 10x - 17)$. It is known that $\int_{-3}^{-1} g(x)dx = -4.8$ and $\int_{-3}^{4} g(x)dx = 11.2$.

(a) Find the value of $\int_{4}^{8} f(x)dx$, or explain why the integral does not exist.

(b)

(i) Find the value of $\int_{-1}^{4} g(x)dx$. Show the work that leads to your answer.

(ii) Find the value of $\int_{-1}^{4} (2g(x) - 4f(x))dx$. Show the work that leads to your answer.

(c) Let $h(x) = \begin{cases} g(x) & \text{for } x \leq -1 \\ f(x) + b & \text{for } x > -1 \end{cases}$. Find the value of b for which $\int_{-3}^{4} h(x)dx = 14.2$.