1) Which of the following is the midpoint Riemann sum approximation of $\int_4^6 \sqrt{x^3+1}dx$ using 4 subintervals of equal width?

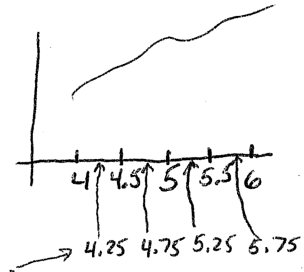
$\Delta x = \frac{6-4}{4} = \frac{2}{4} = \frac{1}{2}$

(A) $\frac{1}{4}\left(\sqrt{4.25^3+1}+\sqrt{4.75^3+1}+\sqrt{5.25^3+1}+\sqrt{5.75^3+1}\right)$

(B) $\frac{1}{2}\left(\sqrt{4.25^3+1}+\sqrt{4.75^3+1}+\sqrt{5.25^3+1}+\sqrt{5.75^3+1}\right)$

(C) $\frac{1}{4}\left(\frac{\sqrt{4^3+1}+\sqrt{4.5^3+1}}{2}+\frac{\sqrt{4.5^3+1}+\sqrt{5^3+1}}{2}+\frac{\sqrt{5^3+1}+\sqrt{5.5^3+1}}{2}+\frac{\sqrt{5.5^3+1}+\sqrt{6^3+1}}{2}\right)$

(D) $\frac{1}{2}\left(\frac{\sqrt{4^3+1}+\sqrt{4.5^3+1}}{2}+\frac{\sqrt{4.5^3+1}+\sqrt{5^3+1}}{2}+\frac{\sqrt{5^3+1}+\sqrt{5.5^3+1}}{2}+\frac{\sqrt{5.5^3+1}+\sqrt{6^3+1}}{2}\right)$



mp →

$A \approx \frac{1}{2}\left(f(4.25)+f(4.75)+f(5.25)+f(5.75)\right)$

2)

t	1	5	8	13
$R(t)$	1	10	17	32

The differentiable function R is increasing, and the graph of R is concave down. Values of $R(t)$ at selected values of t are given in the table above. The trapezoidal sum $(5-1)\left(\frac{10+1}{2}\right)+(8-5)\left(\frac{17+10}{2}\right)+(13-8)\left(\frac{17+32}{2}\right)$ approximates $\int_1^{13} R(t)dt$. Which of the following statements is true?

(A) The trapezoidal sum is an underestimate for $\int_1^{13} R(t)dt$ because R is increasing.

(B) The trapezoidal sum is an overestimate for $\int_1^{13} R(t)dt$ because R is increasing.

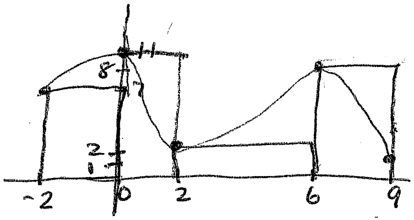
(C) The trapezoidal sum is an underestimate for $\int_1^{13} R(t)dt$ because the graph of R is concave down.

(D) The trapezoidal sum is an overestimate for $\int_1^{13} R(t)dt$ because the graph of R is concave down.

3) Approximate the area between the x-axis and $f(x)$ from $x=-2$ to $x=9$ using a *left Riemann sum* with 4 unequal subdivisions.

x	−2	0	2	6	9
$f(x)$	7	11	2	8	1

The approximate area is $\boxed{68}$ units^2.



$A \approx 2\cdot 7 + 2\cdot 11 + 4\cdot 2 + 3\cdot 8$

$\approx 14 + 22 + 8 + 24$

≈ 68

4) Which of the following definite integrals is equal to $\lim_{n\to\infty}\sum_{k=1}^{n}\frac{10k}{n}\left(\sqrt{1+\frac{5k}{n}}\right)\left(\frac{5}{n}\right)$?

implies function MUST have a variable outside of the radical (so... b or d)

$\Delta x = \frac{5}{n}$ for ALL

(A) $\int_1^6 10\sqrt{x}dx$

(B) $\int_1^6 2x\sqrt{x}dx$ → $f\left(1+\frac{5k}{n}\right)\cdot\frac{5}{n}$

$2\left(1+\frac{5k}{n}\right)\sqrt{1+\frac{5k}{n}}\cdot\frac{5}{n}$

(C) $\int_0^5 10\sqrt{1+x}dx$

(D) $\int_0^5 2x\sqrt{1+x}dx$ → $f\left(0+\frac{5k}{n}\right)\cdot\frac{5}{n}$

$2\left(\frac{5k}{n}\right)\sqrt{1+\frac{5k}{n}}\cdot\frac{5}{n}$

$\frac{10k}{n}\sqrt{1+\frac{5k}{n}}\cdot\frac{5}{n}$

5 Which of the following is a left Riemann sum approximation of $\int_2^8 \cos(x^2)dx$ with n subintervals of equal length? (Careful, this asks for LEFT sum)

Ⓐ $\sum_{k=1}^{n}\left(\cos\left(2+\frac{k-1}{n}\right)^2\right)\frac{1}{n}$

Ⓑ $\sum_{k=1}^{n}\left(\cos\left(\frac{6k}{n}\right)^2\right)\frac{6}{n}$

Ⓒ $\sum_{k=1}^{n}\left(\cos\left(2+\frac{6(k-1)}{n}\right)^2\right)\frac{6}{n}$

Ⓓ $\sum_{k=1}^{n}\left(\cos\left(2+\frac{6k}{n}\right)^2\right)\frac{6}{n}$

$\Delta x = \frac{8-2}{n} = \frac{6}{n}$

$f\left(2+\frac{6(K-1)}{n}\right)\cdot\frac{6}{n}$

$\sum_{i=1}^{n} \cos\left[\left(2+\frac{6(k-1)}{n}\right)^2\right]\cdot\frac{6}{n}$

NOTICE WHEN YOU PUT "1" in for k it says $f(2+0) = f(2)$

so the first height needs to be $f(2)$

since $k=1$ for sigma notation we must use a "$k-1$" to make first height be $f(2)$

6) Which of the following definite integrals are equal to $\lim_{n\to\infty}\sum_{k=1}^{n}\left(-2+\frac{8k}{n}\right)^3\frac{8}{n}$?

I. $\int_{-2}^{6} x^3 dx$ $\longrightarrow f\left(-2+\frac{8k}{n}\right)\cdot\frac{8}{n} \longrightarrow \left(-2+\frac{8K}{n}\right)^3\cdot\frac{8}{n}$ ✓

II. $\int_0^8 (-2+x)^3 dx$ $\longrightarrow f\left(0+\frac{8K}{n}\right)\cdot\frac{8}{n} \longrightarrow \left(-2+\frac{8K}{n}\right)^3\cdot\frac{8}{n}$ ✓

III. $\int_0^1 8(-2+8x)^3 dx$ $\quad f\left(0+\frac{1K}{n}\right)\cdot\frac{1}{n} \longrightarrow 8\left(-2+8\left(\frac{1K}{n}\right)\right)^3\cdot\frac{1}{n} \to \left(-2+\frac{8K}{n}\right)^3\cdot\frac{8}{n}$ ✓

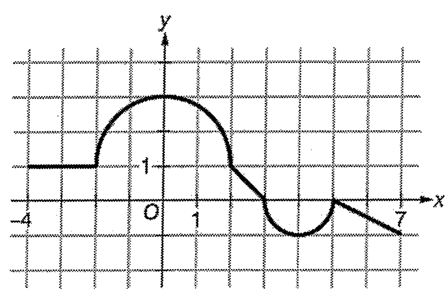
Ⓐ I only

Ⓑ II only

Ⓒ III only

Ⓓ I, II, and III

7)



Graph of f

The graph of the function f on the interval $-4 \le x \le 7$ consists of three line segments and two semicircles, as shown in the figure above. What is the value of $\int_{-4}^{7} f(x)dx$?

Ⓐ $\frac{3}{2}\pi+\frac{3}{2}$

Ⓑ $\frac{3}{2}\pi+\frac{11}{2}$

Ⓒ $\frac{5}{2}\pi+\frac{7}{2}$

Ⓓ $\frac{5}{2}\pi+\frac{15}{2}$

$\int_{-4}^{-2} f(x)dx + \int_{-2}^{2} f(x)dx + \int_{2}^{3} f(x)dx + \int_{3}^{5} f(x)dx + \int_{5}^{7} f(x)dx$

$2\cdot 1 + 4\cdot 1 + \frac{\pi\cdot 2^2}{2} + \frac{1\cdot 1}{2} + -\frac{\pi(1)^2}{2} + -\frac{1\cdot 2}{2}$

$2+4+2\pi+\frac{1}{2}-\frac{\pi}{2}-1$

$\frac{3\pi}{2}+\frac{11}{2}$

8) If $\int_{-2}^{8} (3g(x)+2)dx = 35$ and $\int_{5}^{-2} g(x)dx = -12$, then $\int_{5}^{8} g(x)dx =$

Ⓐ -23

Ⓑ -7

Ⓒ 19

Ⓓ 23

$3\int_{-2}^{8} g(x)dx + \int_{-2}^{8} 2\,dx = 35$

$3\int_{-2}^{8} g(x)dx + 2x\Big|_{-2}^{8} = 35$

$3\int_{-2}^{8} g(x)dx + 16-(-4) = 35$

$3\int_{-2}^{8} g(x) = 15$

$\int_{-2}^{8} g(x) = 5$

Needed First

$\int_{5}^{-2} g(x)dx = -12$

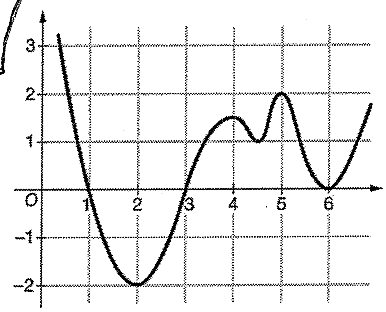
$\int_{-2}^{5} g(x)dx = 12$

Needed First

ANSWER

$\boxed{\begin{array}{l}\int_{-2}^{5} g(x)dx + \int_{5}^{8} g(x)dx = \int_{-2}^{8} g(x)dx \\ 12 + \int_{5}^{8} g(x)dx = 5 \\ \int_{5}^{8} g(x)dx = 5-12 \\ = -7\end{array}}$

9)



Graph of f

← Also $g'(x)$ graph

The graph of the function f is shown above. Let g be the function defined by $g(x) = \int_1^x f(t)dt$. At what values of x in the interval $0.5 < x < 6.5$ does g have a relative maximum?

↖ so... we need g'

Ⓐ 1 only

Ⓑ 3 only

Ⓒ 4 and 5

Ⓓ 1, 3, and 6

$g'(x) = \frac{d}{dx}\int_1^x f(t)dt$

$g'(x) = f(x)$

where $g'(x) = 0$ and crosses from above to below is a rel. MAX on g (This is from unit 3!!)

10) Let f be the function given by $f(x) = \int_6^x (-t^2 - t + 6)dt$. On which of the following intervals is f increasing?

Ⓐ $(-\infty, -\frac{1}{2}]$

Ⓑ $[-3, 2]$

Ⓒ $[-\frac{1}{2}, \infty)$

Ⓓ $(-\infty, -3]$ and $[2, \infty)$

$f'(x) = \frac{d}{dx}\int_6^x -t^2 - t + 6\,dt$

$f'(x) = -x^2 - x + 6$

$0 = -x^2 - x + 6$

$0 = x^2 + x - 6$

$0 = (x+3)(x-2)$

$x+3=0 \quad x-2=0$

$x=-3 \quad x=2$

f': $-$ on $(-\infty,-3)$, $+$ on $(-3,2)$, $-$ on $(2,\infty)$; marks at -3 and 2

NOTE: Be careful to put test values in $f'(x) = -x^2 - x + 6$ to check signs of slopes in the interval

11) If $h(x) = \int_{-1}^{x^3} \sqrt{2+t^2}\,dt$ for $x \geq 0$, then $h'(x) =$ $3x^2\sqrt{2+(x^3)^2}$

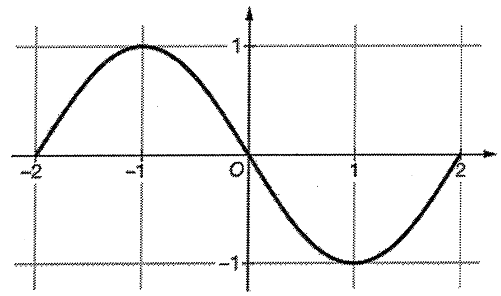
Ⓐ $\sqrt{2+x^2}$

Ⓑ $\sqrt{2+x^6}$

Ⓒ $3x^2\sqrt{2+x^2}$

Ⓓ $3x^2\sqrt{2+x^6}$

12)



Graph of f

← Also $g'(x)$ graph

The graph of the function f defined on the closed interval $[-2, 2]$ is shown above. Let g be defined by $g(x) = \int_0^x f(t)\,dt$. On which of the following intervals is the graph of g both decreasing and concave up?

$g'(x) = f(x)$ so find g'

when g' is negative (below the x-axis) AND increasing (from unit 3)

Ⓐ $(-2, -1)$

Ⓑ $(-1, 0)$

Ⓒ $(0, 1)$

Ⓓ $(1, 2)$

13)

$$f(x) = \begin{cases} 2 & \text{for } 0 \leq x < 2 \\ 3 & \text{for } 2 < x < 3 \\ -1 & \text{for } 3 \leq x \leq 8 \end{cases}$$

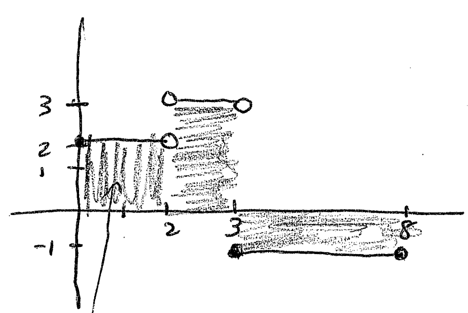
Let f be the piecewise function given above. The value of $\int_0^8 f(x)\,dx$ is

Ⓐ 2

Ⓑ 12

Ⓒ 32

Ⓓ nonexistent



$2 \cdot 2 + 1 \cdot 3 + -(5 \cdot 1)$

$4 + 3 - 5$

$\boxed{2}$

14)

x	2	6	7	9	11	13
$h(x)$	16	−8	−9	−5	7	27
$h'(x)$	−10	−2	0	4	8	12

elected values of the differentiable function h and its first derivative h' are given in the table above. Let f be the function defined by $f(x) = \int_0^x h(t)dt$. What is the least possible number of critical points for f in the interval $2 \le x \le 13$?

so continuous meaning IVT applies

Ⓐ Zero

Ⓑ One

Ⓒ Two

Ⓓ It cannot be determined from the information given.

$f'(x) = \frac{d}{dx}\int_0^x h(t)dt$

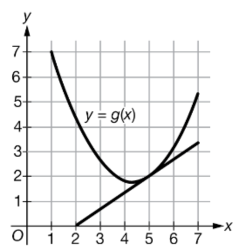
$f'(x) = h(x)$

$f'(x) = 0$?

so. $h(x) = 0$

due to IVT once between [2,6] and a second time between [9,11]

15)



The figure above shows the graph of the differentiable function g and the line tangent to the graph of g at the point $(5, 2)$. Let f be the function given by $f(x) = \int_0^x g(t)dt$. Let h be the function with first derivative given by $h'(x) = x^2 \sin\left(\frac{x}{3}\right)$ for $0 < x < 7$. If the line tangent to the graph of h at $x = a$ is parallel to the line tangent to the graph of f at $x = 5$, what is the value of a?

The figure above shows the graph of the differentiable function g and the line tangent to the graph of g at the point $(5, 2)$. Let f be the function given by $f(x) = \int_0^x g(t)dt$. Let h be the function with first derivative given by $h'(x) = x^2 \sin\left(\frac{x}{3}\right)$ for $0 < x < 7$. If the line tangent to the graph of h at $x = a$ is parallel to the line tangent to the graph of f at $x = 5$, what is the value of a?

Parallel so slopes equal each other

Ⓐ 1.273

Ⓑ 1.857

Ⓒ 2.572

Ⓓ 24.885

$h'(a) = f'(5)$

$a^2 \cdot \sin\left(\frac{a}{3}\right) = 2$

Y_1 Y_2

use Graphing Calculator to find intersection of the two graphs

$a = 1.857$

$f'(x) = \frac{d}{dx}\int_0^x g(t)dt$

$f'(x) = g(x)$

$f'(5) = g(5)$

$f'(5) = 2$

from graph above

16)

x	0	2	4	6
$f(x)$	−22	−6	2	2
$f'(x)$	10	6	2	−2

Selected values of the twice-differentiable function f and its derivative f' are given in the table above. What is the value of $\int_0^6 f'(x)dx$?

(A) −12

(B) 12

(C) 24

(D) 36

$f(x)\Big|_0^6$

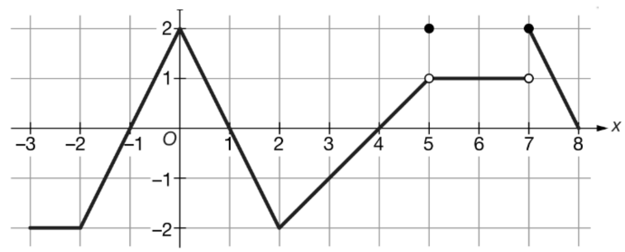
$f(6)-f(0)$

$2-(-22)$

$2+22$

24

17) OPEN ENDED (Calculator NOT Allowed)



Graph of f

The graph of the function f on the closed interval $-3 \le x \le 8$ consists of six line segments and the point $(5, 2)$, as shown in the figure above. The function g is given by $g(x) = \frac{1}{10}(4x^3 + 3x^2 - 10x - 17)$. It is known that $\int_{-3}^{-1} g(x)dx = -4.8$ and $\int_{-3}^{4} g(x)dx = 11.2$.

(a) Find the value of $\int_4^8 f(x)dx$, or explain why the integral does not exist.

$\int_4^7 f(x)dx + \int_7^8 f(x)dx$

$\frac{(3+2)\cdot 1}{2} + \frac{1\cdot 2}{2}$

$\frac{5}{2}+1$

$7/2$

(b)

(i) Find the value of $\int_{-1}^4 g(x)dx$. Show the work that leads to your answer.

$\int_{-3}^{-1} g(x)dx + \int_{-1}^{4} g(x)dx = \int_{-3}^{4} g(x)dx$

$-4.8 + \int_{-1}^{4} g(x)dx = 11.2$

$\int_{-1}^{4} g(x)dx = 11.2+4.8$

$=16$

(ii) Find the value of $\int_{-1}^4 (2g(x) - 4f(x))dx$. Show the work that leads to your answer.

$2\int_{-1}^{4} g(x)\,dx - 4\int_{-1}^{4} f(x)\,dx$

$2\cdot 16 - 4\cdot(-1)$

$32+4 = \boxed{36}$

needed first

$\int_{-1}^{4} f(x)dx = \int_{-1}^{1} f(x)dx + \int_{1}^{4} f(x)dx$

$= \frac{2\cdot 2}{2} + -\left(\frac{3\cdot 2}{2}\right)$

$= 2-3$

$= -1$

(c) Let $h(x) = \begin{cases} g(x) & \text{for } x \le -1 \\ f(x) + b & \text{for } x > -1 \end{cases}$. Find the value of b for which $\int_{-3}^{4} h(x)dx = 14.2$.

$\int_{-3}^{-1} g(x)dx + \int_{-1}^{4} f(x)+b\,dx = \int_{-3}^{4} h(x)dx$

$\int_{-3}^{-1} g(x)dx + \int_{-1}^{4} f(x)dx + \int_{-1}^{4} b\,dx = \int_{-3}^{4} h(x)dx$

$-4.8 + -1 + \int_{-1}^{4} b\,dx = 14.2$

$\int_{-1}^{4} b\,dx = 20$

$bx\Big|_{-1}^{4} = 20$

$4b-(-1b) = 20$

$5b = 20$

$\boxed{b=4}$