Unit 4 Lessons 1-5 Quiz OE Questions /5 Name Key

1) If $F(x)=\int_0^{\sqrt{x}} \cos(t^2)\,dt$, then $F'(4)=$

(3) $F'(x)=\frac{d}{dx}\int_0^{\sqrt{x}}\cos(t^2)dt$

$F'(x)=\cos x\cdot\frac{1}{2}x^{-1/2}$

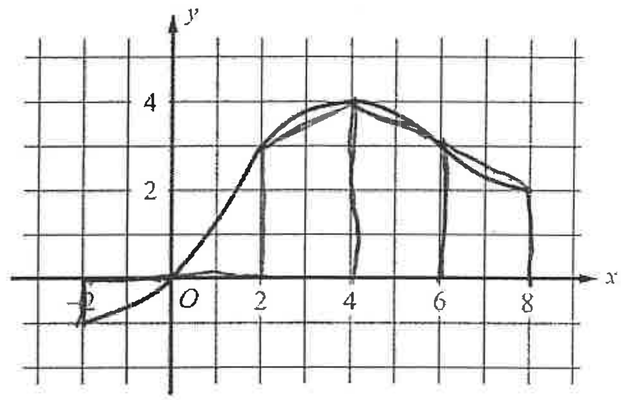
$F'(4)=\cos 4\cdot\frac{1}{2}(4)^{-1/2}$

or

$\cos 4\cdot\frac{1}{4}$

$=\frac{\cos 4}{4}\approx -.163$

2)



The graph of a differentiable function f on the closed interval $[-2,8]$ is shown in the figure above. Find a trapezoidal approximation of $\int_{-2}^{8} f(x)\,dx$ using five subdivisions of length $\Delta x=2$.

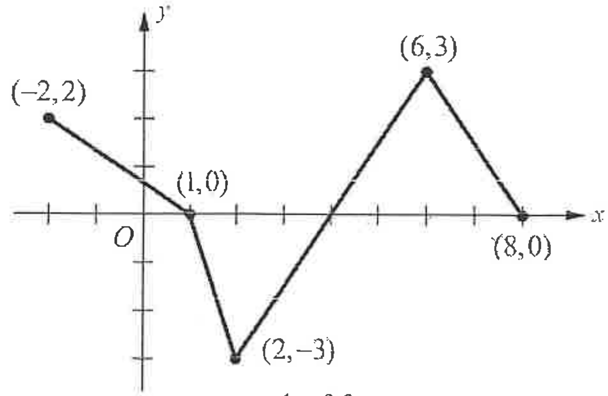
(3) $-\left(\frac{2\cdot 1}{\cancel{2}}\right)+\left(\frac{2\cdot 3}{\cancel{2}}\right)+\frac{(3+4)\cdot\cancel{2}}{\cancel{2}}+\frac{(3+4)\cdot\cancel{2}}{\cancel{2}}+\frac{(3+2)\cdot\cancel{2}}{\cancel{2}}$

$-1+3+7+7+5$

21

6)

3)



graph of f

The graph of f, consisting of four line segments, is shown in the figure above. Let g be the function given by $g(x)=\int_{-2}^{x} f(t)\,dt$.

$g'(x) = \frac{d}{dx}\int_{-2}^{x} f(t)$

(a) Find $g'(1)$.

$g'(x) = f(x)$

(b) Find the x-coordinate for each point of inflection of the graph of g on the interval $-2 < x < 8$.

(c) Find the average rate of change of g on the interval $2 \le x \le 8$.

(d) For how many values of c, where $2 < c < 8$, is $g'(c)$ equal to the average rate found in part (c)? Explain your reasoning.

(2) (a) $g'(1) = f(1)$

$= 0$

(2) (b) Since $g'(x)$ is $f(x)$ graph, mins / maxs on $f(x)$ graph

so $x = 2$ & $x = 6$

(3) (c) $\dfrac{g(8) - g(2)}{8-2} = \boxed{\dfrac{1}{2}}$

$\dfrac{\frac{9}{2} - \frac{3}{2}}{6} = \dfrac{3}{6}$

$g(8) = \int_{-2}^{8} f(t)dt = \frac{3\cdot 2}{2} - \frac{3\cdot 3}{2} + \frac{4\cdot 3}{2}$

$= 3 - \frac{9}{2} + 6 = \frac{9}{2}$

$g(2) = \int_{-2}^{2} f(t)dt = \int_{-2}^{1} f(t)dt + \int_{1}^{2} f(t)dt +$

$= 3 - \frac{1\cdot 3}{2}$

$= \frac{3}{2}$

(2) (d) 2 times as $g'(x)$ is $f(x)$ graph above and it has a y-coord. of $1/2$ 2 times in this interval

9

Unit 4 Lessons 1-5 Quiz OE Questions 15 Name Key

1) Let $F(x)=\int_0^x\left[\sin 2t+t^2\right]dt$ on the closed interval $[0, 2\pi]$.

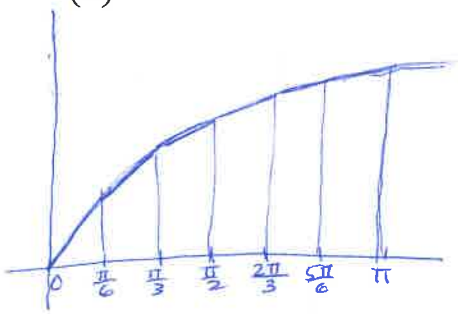
(a) Approximate $F(\pi)$ using six inscribed trapezoids.

(b) Find $F'(2\pi)$.

$F(\pi) = \int_0^{\pi} \sin(2t)+t^2\,dt \qquad \Delta x = \frac{\pi-0}{6} = \frac{\pi}{6}$

(3)

(a)



$\frac{1}{2}\cdot\frac{\pi}{6}\left(f(0)+2f(\tfrac{\pi}{6})+2f(\tfrac{\pi}{3})+2f(\tfrac{\pi}{2})+2f(\tfrac{2\pi}{3})+2f(\tfrac{5\pi}{6})+f(\pi)\right)$

$=\frac{\pi}{12}\left(0+2\cdot\left(\frac{\sqrt3}{2}+\frac{\pi^2}{36}\right)+2\cdot\left(\frac{\sqrt3}{2}+\frac{\pi^2}{9}\right)+\frac{\pi^2}{2}+2\left(-\frac{\sqrt3}{2}+\frac{4\pi^2}{9}\right)+2\cdot\left(-\frac{\sqrt3}{2}+\frac{25\pi^2}{36}\right)+\pi^2\right)$

$=\frac{\pi}{12}\left(\cancel{\sqrt3}+\frac{\pi^2}{18}+\cancel{\sqrt3}+\frac{2\pi^2}{9}+\frac{\pi^2}{2}+-\cancel{\sqrt3}+\frac{8\pi^2}{9}+-\cancel{\sqrt3}+\frac{25\pi^2}{18}+\pi^2\right)$

$=\frac{\pi}{12}\left(\frac{\pi^2}{18}+\frac{2\pi^2}{9}+\frac{\pi^2}{2}+\frac{8\pi^2}{9}+\frac{25\pi^2}{18}+\pi^2\right)$

$\boxed{=\frac{\pi}{12}\left(\pi^2+\frac{\pi^2}{2}+\frac{23\pi^2}{9}\right)}$

(b)

$F'(x)=\frac{d}{dx}\int_0^x \sin(2t)+t^2\,dt$

(2)

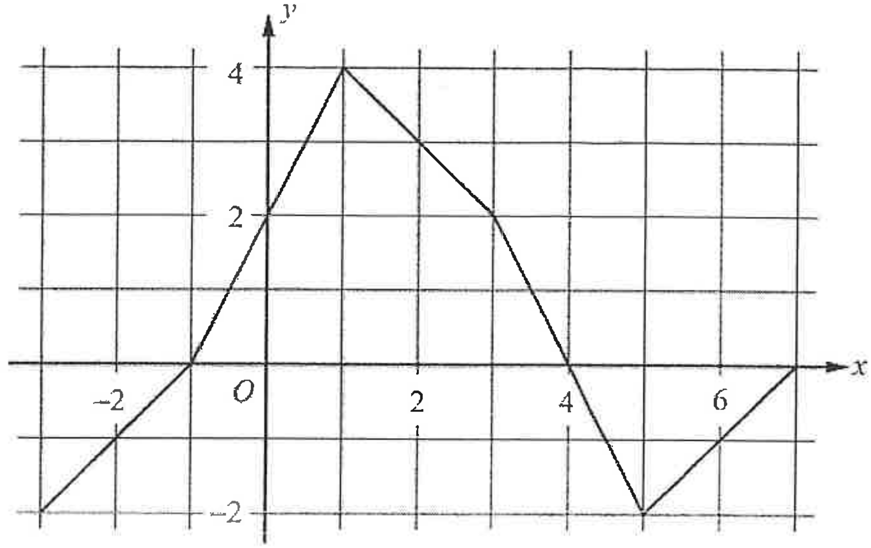
$F'(x) = \sin(2x)+x^2$

$F'(2\pi) = \sin(4\pi)+(2\pi)^2$

$= 0 + 4\pi^2$

$= 4\pi^2$

2)



Graph of f

The graph of the function f above consists of five line segments. Let g be the function given by $g(x) = \int_{-3}^{x} f(t)\, dt$.

(a) Find $g(2)$, $g'(2)$ and $g''(2)$.

(b) Find the x-coordinate of each point of inflection of the graph of g on the open interval $-3 < x < 7$. Justify your answer.

(c) On what interval is g increasing?

(d) Find the average rate of change of g on the interval $0 \le x \le 4$

(3) (a) $g(2) = \int_{-3}^{2} f(t)\,dt = \frac{11}{2}$

$-\left(\frac{2\cdot 2}{2}\right) + \frac{2\cdot 4}{2} + \frac{(4+3)\cdot 1}{2}$

$-2 + 4 + 7/2$

$g'(2) = g'(x) = f(x)$

$g'(2) = f(2)$

$= 3$

$g''(2) = g''(x) = f'(x)$

$g''(2) = f'(2)$

$= -1$

(b) Since $g'(x) = f(x)$ where $f(x)$ has min/max

(2) $\boxed{x = 1 \text{ or } x = 5}$

(c) Since $g'(x) = f(x)$ where $f(x)$ is positive (above the x-axis)

(2) $[-1, 4]$

(d) (3) $\frac{g(4) - g(0)}{4 - 0} = \frac{g(4) - g(0)}{4} = \frac{9 - (-1)}{4}$

$= \frac{10}{4}$

$\boxed{= \frac{5}{2}}$

$g(4) = \int_{-3}^{4} f(t)\,dt = -2 + 4 + \frac{(4+2)\cdot 2}{2} + \frac{1\cdot 2}{2}$

$-2 + 4 + 6 + 1$

$= 9$

$g(0) = \int_{-3}^{0} f(t)\,dt = -2 + \frac{1\cdot 2}{2}$

$-2 + 1$

$= -1$

Unit 4 Lessons 1-5 Quiz OE Questions 15 Name Key

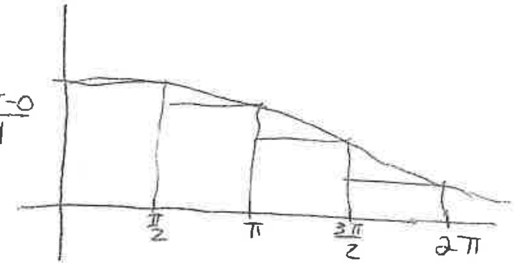
1) Let $F(x) = \int_0^x \left[\cos\left(\frac{t}{2}\right) + \left(\frac{3}{2}\right)\right] dt$ on the closed interval $[0, 2\pi]$.

(a) Approximate $F(2\pi)$ using four right hand rectangles.

(b) Find $F'(2\pi)$.

$\Delta x = \frac{2\pi - 0}{4}$

$\Delta x = \frac{\pi}{2}$



(a) $F(2\pi) = \int_0^{2\pi} \left(\cos\left(\frac{t}{2}\right) + \frac{3}{2}\right)$

$= \frac{\pi}{2} \cdot \left(\cos\frac{\frac{\pi}{2}}{2} + \frac{3}{2} + \cos\frac{\pi}{2} + \frac{3}{2} + \cos\frac{\frac{3\pi}{2}}{2} + \frac{3}{2} + \cos\frac{2\pi}{2} + \frac{3}{2}\right)$

(3) $= \frac{\pi}{2}\left(\cos\frac{\pi}{4} + \cos\frac{\pi}{2} + \cos\frac{3\pi}{4} + \cos\pi + 4\cdot\left(\frac{3}{2}\right)\right)$

$= \frac{\pi}{2}\left(\frac{\sqrt{2}}{2} + 0 + \frac{-\sqrt{2}}{2} + -1 + 6\right)$

$= \frac{\pi}{2}(5)$

$= \boxed{\frac{5\pi}{2}}$

(b) $F'(x) = \frac{d}{dx}\int_0^x \cos\frac{t}{2} + \frac{3}{2}\, dt$

$F'(x) = \cos\frac{x}{2} + \frac{3}{2}$

(2) $F'(2\pi) = \cos\frac{2\pi}{2} + \frac{3}{2}$

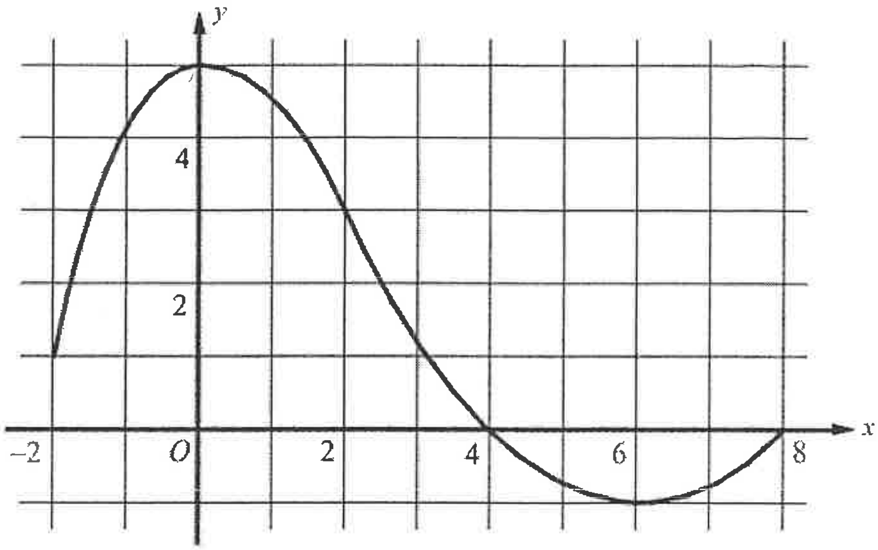
$= \cos\pi + \frac{3}{2}$

$= -1 + \frac{3}{2}$

$= \boxed{\frac{1}{2}}$

5)

2)



Graph of f

The graph of a differentiable function f on the closed interval $[-2,8]$ is shown in the figure above.

The graph of f has a horizontal tangent line at $x=0$ and $x=6$. Let $h(x)=-3+\int_0^x f(t)\,dt$ for $-2 \le x \le 8$.

$h'(x) = 0 + f(x)$
$h'(x) = f(x)$
$h''(x) = f'(x)$

(a) Find $h(0)$, $h'(0)$, and $h''(0)$.

(b) On what interval is h decreasing? Justify your answer.

(c) On what intervals does the graph of h concave up? Justify your answer.

(d) Find a trapezoidal approximation of $\int_{-2}^{8} f(t)\,dt$ using five subintervals of length $\Delta t = 2$.

(3) (a) $h(0) = -3 + \int_0^0 f(t)dt$ $\quad$ $-3+0$ $\quad$ $= \boxed{-3}$ $\qquad$ $h'(0) = f(0) = \boxed{5}$ $\qquad$ $h''(0) = f'(0) = \boxed{0}$

(2) (b) $h'(x) = f(x)$ so when $f(x)$ is negative (below x-axis)

$[4,8]$

(2) (c) $h'(x) = f(x)$ so when $f(x)$ INCREASES

$[-2,0] \cup [6,8]$

(3) (d) "h" from formula

$\frac{2}{2}\left(1 + 2(5) + 2(3) + 2(0) + 2(-1) + 0\right)$

$1 \cdot (1 + 10 + 6 + 0 - 2 + 0)$

$1(15) \quad = \boxed{15}$

or $\frac{(1+5)\cdot 2}{2} + \frac{(5+3)\cdot 2}{2} + \frac{(3+0)\cdot 2}{2} - \frac{(0+1)\cdot 2}{2} - \frac{(1+0)\cdot 2}{2}$

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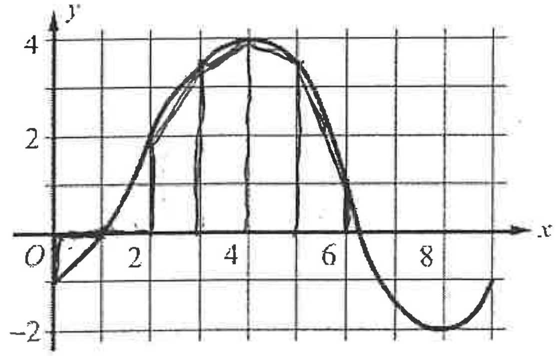
1)



graph of g

(a) The graph of the function g, shown in the figure above, has horizontal tangents at $x = 4$ and $x = 8$. If $f(x) = \int_0^{\sqrt{x}} g(t)\,dt$, what is the value of $f'(4)$?

(3)

$$f'(x) = \frac{d}{dx}\int_0^{\sqrt{x}} g(t)\,dt$$

$$f'(x) = g(\sqrt{x}) \cdot \tfrac{1}{2}x^{-1/2}$$

$$f'(4) = g(\sqrt{4}) \cdot \tfrac{1}{2}(4)^{-1/2}$$

$$= g(2) \cdot \tfrac{1}{4}$$

$$= 2 \cdot \tfrac{1}{4}$$

$$= \boxed{\tfrac{1}{2}}$$

(b) Find a trapezoidal approximation of $\int_0^6 g(x)\,dx$ using six trapezoids.

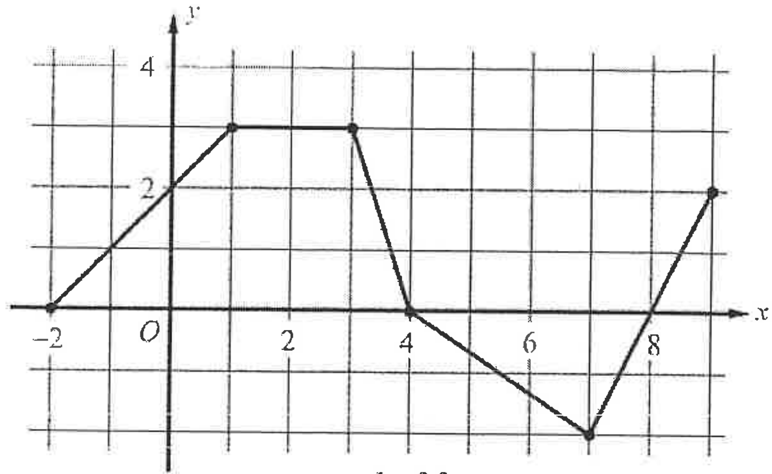
$$-\left(\frac{1 \cdot 1}{2}\right) + \left(\frac{1 \cdot 2}{2}\right) + \frac{(2+3.5) \cdot 1}{2} + \frac{(3.5+4) \cdot 1}{2} + \frac{(3.5+4) \cdot 1}{2} + \frac{(3.5+1) \cdot 1}{2}$$

(3)

$$-\tfrac{1}{2} + 1 + 2.75 + 3.75 + 3.75 + 2.25$$

$$\boxed{13}$$

2)



graph of f

Let g be the function given by $g(x) = \int_{-2}^{x} f(t)\,dt$. The graph of the function f, shown above, consists of five line segments.

$g'(x) = \frac{d}{dx}\int_{-2}^{x} f(t)\,dt$

$g'(x) = f(x)$

(a) Find $g(0)$, $g'(0)$ and $g''(0)$.

(b) For what values of x, in the open interval $(-2,9)$, is the graph of g concave up?

(c) For what values of x, in the open interval $(-2,9)$, is g increasing?

(d) Find the average rate of change for g on $2 \le x \le 8$.

(2) (a) $g(0) = \int_{-2}^{0} f(t)dt$ $\quad \frac{2 \cdot 2}{2} = \boxed{2}$

$g'(0) = g'(x) = f(x)$ $\quad f(0) = \boxed{2}$

$g''(0) = g''(x) = f'(x)$ $\quad f'(0) = \boxed{1}$

(2) (b) Since $f(x)$ is $g'(x)$ graph, it is when graph above increases, So... $[-2,1] \cup [7,9]$

(2) (c) g increases where $g'(x)$ is POSITIVE. So, $[-2,4] \cup [8,9]$ because graph above is Above axis

(3) (d) $\frac{g(8) - g(2)}{8-2} = \boxed{\frac{1}{12}}$

$g(8) = \int_{-2}^{8} f(t)dt = \frac{(6+2)\cdot 3}{2} - \frac{4 \cdot 2}{2}$

$= 12 - 4$

$= 8$

$\frac{8 - \frac{15}{2}}{6} = \frac{16-15}{12} = \frac{1}{12}$

$g(2) = \int_{-2}^{2} f(t)dt = \frac{(4+1)\cdot 3}{2}$

$= \frac{15}{2}$