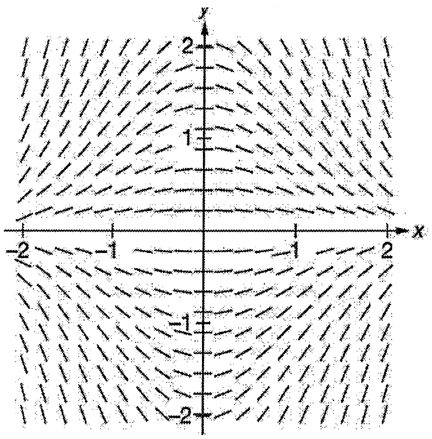
Unit 4 AP Classroom Practice for Lessons 6-8

1)



Shown above is the slope field for which of the following differential equations?

(A) $\frac{dy}{dx} = -(x+y)$ — (0,1) should have slope of "−1" for this one but slope field shows "0"

(B) $\frac{dy}{dx} = -xy$ — (0,0) should have a slope of "0" but apparently is undefined as seen on slopefield

(C) $\frac{dy}{dx} = -x^2y$ — (−1,1) should have a slope of NEGATIVE one but slopefield shows a positive slope

(D) $\frac{dy}{dx} = -\frac{x}{y}$ — NO slopes drawn when $y=0$. Also, slope ALWAYS zero when $x=0$ (other than $y=0$ too)

2) What is the general solution to the differential equation $\frac{dy}{dx} = \frac{x-1}{3y^2}$ for $y > 0$?

(A) $y = \frac{1}{\left(-\frac{3}{2}x^2+3x+C\right)}$

(B) $y = \left(e^{\left(\frac{3x^2}{2}-3x+C\right)}\right)^{\frac{1}{2}}$

(C) $y = (x^2-x)^{\frac{1}{3}}+C$

(D) $y = \left(\frac{x^2}{2}-x+C\right)^{\frac{1}{3}}$

$3y^2dy = x-1\,dx$

$\int 3y^2dy = \int x-1\,dx$

$\frac{\cancel{3}y^3}{\cancel{3}} = \frac{x^2}{2}-x+C$

$y^3 = \frac{1}{2}x^2-x+C$

$y = \sqrt[3]{\frac{x^2}{2}-x+C}$

3)

What is the general solution to the differential equation $\frac{dy}{dx} = \sqrt{y} - \sqrt{y}\sin x$ for $y > 0$?

(A) $y = Ce^{2x+2\cos x}$

(B) $y = \left(\frac{3}{2}(x + \cos x + C)\right)^{\frac{2}{3}}$

(C) $y = \frac{1}{4}(x + \cos x)^2 + C$

(D) $y = \frac{1}{4}(x + \cos x + C)^2$

$\frac{dy}{dx} = \sqrt{y}(1 - \sin x)$

$\frac{1}{\sqrt{y}} dy = (1 - \sin x)\, dx$

$\int y^{-1/2} dy = \int 1 - \sin x \, dx$

$2y^{1/2} = x + \cos x + C$

$y^{1/2} = \frac{x + \cos x + C}{2}$

$y = \left(\frac{x + \cos x + C}{2}\right)^2$

$y = \frac{1}{4}(x + \cos x + C)^2$

4)

Let $y = f(x)$ be the particular solution to the differential equation $\frac{dy}{dx} = \frac{1}{2y+1}$ with the initial condition $y(0) = 0$. Which of the following gives an expression for $f(x)$ and the domain for which the solution is valid?

(A) $y = \frac{e^{2x}-1}{2}$ for all x

(B) $y = \tan x$ for $-\frac{\pi}{2} < x < \frac{\pi}{2}$

(C) $y = \sqrt{x}$ for $x \geq 0$

(D) $y = \frac{-1+\sqrt{1+4x}}{2}$ for $x > -\frac{1}{4}$

$2y + 1\, dy = 1\, dx$

$\int 2y + 1\, dy = \int 1\, dx$

$\frac{2y^2}{2} + y = x + C$

$y^2 + y = x + C$

$0 + 0 = 0 + C$

$0 = C$

$y^2 + y = x$

$y^2 + y - x = 0$

$\Downarrow$

use quad formula

$a = 1, \; b = 1 \; c = -x$

$y = \frac{-1 \pm \sqrt{1 - 4(1)(-x)}}{2(1)}$

$y = \frac{-1 + \sqrt{1+4x}}{2}$

(0,0) has to be a solution

5) Graphing Calculator Needed

Let f be the function given by $f(x) = \sin(2x)\cos(1 + x)$. What is the average value of f on the closed interval $1 \leq x \leq 3$?

(A) 0.739

(B) 0.369

(C) 0.281

(D) -0.098

$\frac{1}{3-1}\int_1^3 f(x)\, dx$

$\frac{1}{2}(.738978)$

$= .369$

6)

The average value of a function f over the interval $[-2, 3]$ is -6, and the average value of f over the interval $[3, 5]$ is 20. What is the average value of f over the interval $[-2, 5]$?

(A) 2

(B) 7

(C) $\frac{10}{7}$

(D) 5

$$\frac{1}{3-(-2)} \int_{-2}^{3} f(x) = -6 \qquad \frac{1}{5-3} \int_{3}^{5} f(x)\,dx = 20$$

$$\frac{1}{5} \cdot \int_{-2}^{3} f(x)\,dx = -6 \qquad \frac{1}{2} \int_{3}^{5} f(x)\,dx = 20$$

$$\boxed{\frac{10}{7}} = \frac{1}{7}\int_{-2}^{5} f(x)\,dx \qquad \int_{-2}^{3} f(x)\,dx = -30 \qquad \int_{3}^{5} f(x)\,dx = 40$$

$$\int_{-2}^{3} f(x)\,dx + \int_{3}^{5} f(x)\,dx = \int_{-2}^{5} f(x)\,dx$$

$$-30 + 40 = \int_{-2}^{5} f(x)\,dx$$

$$10 = \int_{-2}^{5} f(x)\,dx$$

7)

The intensity of radiation at a distance x meters from a source is modeled by the function R given by $R(x) = \frac{k}{x^2}$, where k is a positive constant. Which of the following gives the average intensity of radiation between 10 meters and 50 meters from the source?

(A) $\frac{k}{900}$

(B) $-\frac{k}{50} + \frac{k}{10}$

(C) $\frac{1}{40}\left(-\frac{k}{50} + \frac{k}{10}\right)$

(D) $\frac{1}{2}\left(\frac{k}{100} + \frac{k}{2500}\right)$

AVG VALUE of $R(x)$ over $[10, 50]$

$$\frac{1}{50-10} \int_{10}^{50} k \cdot x^{-2}\,dx$$

$$\frac{k}{40}\left[-x^{-1} \Big|_{10}^{50}\right]$$

$$\frac{k}{40}\left[-50^{-1} - (-10^{-1})\right] \rightarrow \frac{k}{40}\left[-\frac{1}{50} + \frac{1}{10}\right]$$

8) GRAPHING CALCULATOR NEEDED!

A cup of coffee is poured, and the temperature is measured to be 120 degrees Fahrenheit. The temperature of the coffee then decreases at a rate modeled by $r(t) = 55e^{-0.03t^2}$ degrees Fahrenheit per minute, where t is the number of minutes since the coffee was poured. What is the temperature of the coffee, in degrees Fahrenheit, at time $t = 1$ minute?

(A) 53.4°F

(B) 54.5°F

(C) 65.5°F

(D) 66.6°F

$T(0) = 120$

$$T(t) = 120 - \int_0^t r(t)\,dt$$

$$T(1) = 120 - \int_0^1 r(t)\,dt$$

$$T(1) = 120 - 54.455$$ from calculator

$$T(1) = 65.545$$

10 (d)

$D(t) = 12 + \int_0^t m(t)dt - \int_0^t c(t)dt$

$D'(t) = m(t) - c(t)$

$0 = m(t) - c(t)$ ← use calculator

$D'(t)$: $+$ (4.584) $-$ (16.715) $+$

rel MAX at $t = 4.584$

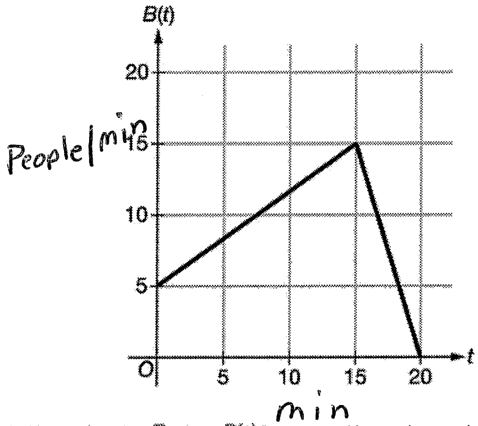
so... EXTREME VALUE THM

$D(0) = 12$ meters apart

$D(4.584) = 365.527$ meters apart

$D(20) = 27.010$ meters apart

9)



The rate at which people arrive at a theater box office is modeled by the function B, where $B(t)$ is measured in people per minute and t is measured in minutes. The graph of B for $0 \leq t \leq 20$ is shown in the figure above. Which of the following is closest to the number of people that arrive at the box office during the time interval $0 \leq t \leq 20$?

Ⓐ 15.

Ⓑ 38

Ⓒ 150

Ⓓ 188

$\int_0^{20} B(t)\,dt$ (use geometry)

$\int_0^{15} B(t)\,dt + \int_{15}^{20} B(t)\,dt$

$\frac{(5+15)\cdot 15}{2} + \frac{5 \cdot 15}{2}$

$\frac{\cancel{20}^{10} \cdot 15}{\cancel{2}_1} + \frac{5 \cdot 15}{2}$

$150 + \frac{75}{2}$ $\qquad \frac{300+75}{2} = \frac{375}{2}$

≈ 187.5

10) Open Ended: GRAPHING CALCULATOR NEEDED!

Mary and Chance walk in the same ~~direction along a straight path~~. For $0 \leq t \leq 20$, Mary's velocity at time t is given by $M(t) = \frac{6010}{t^2 - 3t + 50.5}$ and Chance's velocity at time t is given by $C(t) = 8.5t^3 e^{-0.45t}$. Both $M(t)$ and $C(t)$ are positive for $0 \leq t \leq 20$ and are measured in meters per minute, and t is measured in minutes. Mary is 12 meters ahead of Chance at time $t = 0$, and Mary remains ahead of Chance for $0 < t \leq 20$.

→ So... we do not need absolute value when calculating distance using an integral

(a) Find the value of $\frac{1}{10}\int_5^{15} M(t)dt$. Using correct units, interpret the meaning of $\frac{1}{10}\int_5^{15} M(t)dt$ in the context of the problem.

(b) At time $t = 10$, is Mary speeding up or slowing down? Give a reason for your answer.

(c) Is the distance between Mary and Chance at time $t = 18$ increasing or decreasing? Give a reason for your answer.

(d) What is the maximum distance between Mary and Chance over the time interval $0 \leq t \leq 20$? Justify your answer.

(a) $\frac{1}{10}\int_5^{15} m(t)dt$ finds average velocity for Mary from $t = 5$ to $t = 15$ which equals 54.409 met/min.

UNIT 3 { (b) $m(10) = $ "+" 49.9 which is her velocity. $m'(10) = $ "−" 7.03 which is her acceleration. Since velocity and acceleration are opposite sign, Mary is slowing down.

(c) MARY'S DISTANCE: $12 + \int_0^t m(t)dt$ Chase's Distance: $\int_0^t c(t)dt$

Distance between them: $12 + \int_0^t m(t)dt - \int_0^t c(t)dt$ so $D(t) = 12 + \int_0^t m(t)dt - \int_0^t c(t)dt$

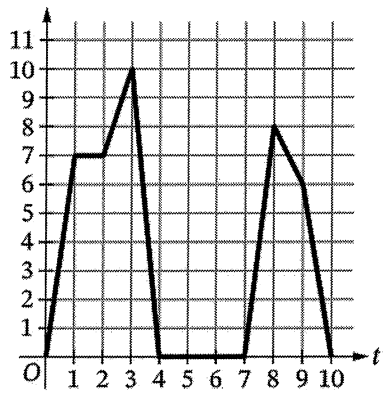
$D'(t) = 0 + m(t) - c(t)$

$D'(18) = 0 + 18.752 - 15.047$

$D'(18) = $ "+" 3.705 so distance between them is INCREASING since $D'(18)$ is POSITIVE!

11) The Iditarod dog sled race takes place annually in Alaska. The velocity of one dog sled, $v(t)$, measured in miles per hour, was tracked for a 10-hour period from 12 p.m. ($t = 0$) to 10 p.m. ($t = 10$).

mph



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The graph of the dog sled's velocity during this 10-hour time period is shown above.

(a) Using correct units, interpret the meaning of $\int_0^{10} v(t)\,dt$ in the context of this problem.

Find the value of $\int_0^{10} v(t)\,dt$. Describe how you found your answer.

(b) What is the average velocity of the dog sled on the interval $0 \le t \le 4$? Show work that leads to your answer.

(c) Assume positive velocity represents movement away from the dog's home. If the dog sled's position at 12 p.m. ($t = 0$) was 20 miles away from home, what would be its position at 10 p.m. ($t = 10$)? Show work that leads to your answer.

(a) $\int_0^{10} v(t)dt$ is the total position change in miles from 12pm to 10pm

$$\int_0^1 v(t)dt + \int_1^2 v(t)dt + \int_2^3 v(t)dt + \int_3^4 v(t)dt + \int_4^7 v(t)dt + \int_7^8 v(t)dt + \int_8^9 v(t)dt + \int_9^{10} v(t)dt$$

$$\frac{1\cdot 7}{2} + 1\cdot 7 + \frac{(7+10)\cdot 1}{2} + \frac{10\cdot 1}{2} + 0 + \frac{1\cdot 8}{2} + \frac{(8+6)\cdot 1}{2} + \frac{1\cdot 6}{2}$$

$$\frac{7}{2} + 7 + \frac{17}{2} + 5 + 0 + 4 + 7 + 3 = 38 \text{ miles}$$

(b) $\frac{1}{4-0}\int_0^4 v(t)dt = \frac{1}{4}\cdot 24 = 6$ mph

(c) $P(t) = 20 + \int_0^t v(t)\,dt$

$P(10) = 20 + \int_0^{10} v(t)\,dt$

$= 20 + 38$

$= 58$ miles from home at 10 p.m.