AP Classroom U-Substitution Practice(Unit 5 Lessons 1-4)

1) $\int_0^1 (x^3+x)(x^4+2x^2+9)^{\frac{1}{2}}\,dx =$

$u = x^4+2x^2+9$

$\frac{du}{dx} = 4x^3+4x$

$= 4(x^3+x)$

Ⓐ $\frac{1}{6}$

Ⓑ $\frac{2}{3}$

Ⓒ $4\sqrt{3}-\frac{9}{2}$

Ⓓ $16\sqrt{3}-18$

$\int_9^{12} \cancel{(x^3+x)} \cdot u^{1/2} \cdot \frac{1}{4\cancel{(x^3+x)}}\,du$

$\int_9^{12} \frac{1}{4} u^{1/2}\,du = \frac{1}{6}u^{3/2}\Big|_9^{12}$

$\frac{1}{6}(\sqrt{12})^3 - \frac{1}{6}(\sqrt{9})^3$

$\frac{1}{6}(2\sqrt{3})^3 - \frac{27}{6}$

$\frac{1}{6}\cdot 8\cdot 3\sqrt{3} - \frac{27}{6}$

$4\sqrt{3}-\frac{9}{2}$

2) $\int \frac{3x^2+4x+1}{3x^3+6x^2+3x+5}\,dx =$

$u = 3x^3+6x^2+3x+5$

$\frac{du}{dx} = 9x^2+12x+3$

$= 3(3x^2+4x+1)$

Ⓐ $-\frac{9x^2+12x+3}{(3x^3+6x^2+3x+5)^2}+C$

Ⓑ $\frac{x^3+2x^2+x}{\frac{3}{4}x^4+2x^3+\frac{3}{2}x^2+5x}+C$

Ⓒ $\frac{1}{3}\ln|3x^3+6x^2+3x+5|+C$

Ⓓ $\ln|3x^3+6x^2+3x+5|+C$

$\int \frac{\cancel{3x^2+4x+1}}{u} \cdot \frac{1}{3\cancel{(3x^2+4x+1)}}\,du$

$\frac{1}{3}\int \frac{1}{u}\,du$

$\frac{1}{3}\ln|u|+C$

$\frac{1}{3}\ln|3x^3+6x^2+3x+5|+C$

3) $\int \frac{12x^2}{2x+1}\,dx =$

(A) $2x^3 \ln|2x+1| + C$

(B) $4x^3 \ln|2x+1| + C$

(C) $3x^2 - 3x + \frac{3}{2}\ln|2x+1| + C$

(D) $3x^2 - 3x + 3\ln|2x+1| + C$

$$\begin{array}{r} 6x - 3 \\ 2x+1 \,\big)\, 12x^2 + 0x + 0 \\ -(12x^2+6x) \\ \hline -6x + 0 \\ -(-6x-3) \\ \hline 3 \end{array}$$

$$= \int 6x - 3 + \frac{3}{2x+1}\,dx$$

$$= 3x^2 - 3x + \int \frac{3}{2x+1}\,dx$$

$$= 3x^2 - 3x + 3\int \frac{1}{u}\cdot\frac{1}{2}\,du \quad u = 2x+1 \quad \frac{du}{dx} = 2$$

$$= 3x^2 - 3x + \frac{3}{2}\int \frac{1}{u}\,du$$

$$= 3x^2 - 3x + \frac{3}{2}\ln|u| + C$$

$$= 3x^2 - 3x + \frac{3}{2}\ln|2x+1| + C$$

4) $\int_3^5 \frac{2x^2+x+4}{x-1}\,dx =$

(A) $\frac{139}{9} - 23$

(B) $\frac{\frac{605}{6} - \frac{69}{2}}{\frac{15}{2} - \frac{3}{2}}$

(C) $22 + 7(\ln 4 - \ln 2)$

(D) $24 + 8(\ln 4 - \ln 2)$

$$\begin{array}{r} 2x + 3 \\ x-1 \,\big)\, 2x^2 + x + 4 \\ -(2x^2 - 2x) \\ \hline 3x + 4 \\ -(3x - 3) \\ \hline 7 \end{array}$$

$$= \int_3^5 2x + 3 + \frac{7}{x-1}\,dx$$

$7\int \frac{1}{u}\,du$ $\quad u = x-1 \quad \frac{du}{dx} = 1$

$$x^2 + 3x + 7\ln|x-1| \Big|_3^5$$

$$(25 + 15 + 7\ln 4) - (9 + 9 + 7\ln 2)$$

$$40 - 18 + 7\ln 4 - 7\ln 2$$

$$22 + 7(\ln 4 - \ln 2)$$

5) Using the substitution $u = 4x - 3$, $\int x(4x-3)^{10}dx$ is equivalent to which of the following?

$u = 4x-3$

$\frac{du}{dx} = 4$

Ⓐ $\int u^{10}du$

$= \int x \cdot u^{10} \cdot \frac{1}{4}\,du$

Ⓑ $\frac{1}{4}\int u^{10}du$

$= \int \frac{u+3}{4} \cdot u^{10} \cdot \frac{1}{4}\,du$

if $u = 4x-3$ then $x = \frac{u+3}{4}$

Ⓒ $\frac{1}{4}\int (u^{11} + 3u^{10})du$

$= \frac{1}{4}\int \left(\frac{u}{4} + \frac{3}{4}\right)u^{10}\,du$

$= \frac{1}{4}\int \frac{1}{4}(u+3)\cdot u^{10}\,du$

Ⓓ $\frac{1}{16}\int (u^{11} + 3u^{10})du$

$= \frac{1}{16}\int u^{11} + 3u^{10}\,du$

6) Which of the following are equivalent to $\int_0^5 \frac{3x+11}{x+2}dx$?

I. $\cancel{\dfrac{\int_0^5 (3x+11)dx}{\int_0^5 (x+2)dx}}$

NOT TRUE (This is NOT a property of integrals)

II. $\int_0^5 \left(3 + \frac{5}{x+2}\right)dx$

III. $\int_2^7 \left(3 + \frac{5}{u}\right)du$

$$\begin{array}{r} 3 \\ x+2\overline{)3x+11} \\ -(3x+6) \\ \hline 5 \end{array}$$

Ⓐ I only

Ⓑ II only

$= \int_0^5 3 + \frac{5}{x+2}\,dx \leftarrow \text{II}$

Ⓒ III only

$u = x+2$

$\frac{du}{dx} = 1$

$= \int_2^7 3 + \frac{5}{u}\,du \leftarrow \text{III}$

Ⓓ II and III only

7) Which of the following is equivalent to $\int_1^2 \frac{x^7 - 4x^3 + 6}{x^2} dx$?

$= \int_1^2 x^{-2} \cdot (x^7 - 4x^3 + 6)$

$= \int_1^2 x^5 - 4x + 6x^{-2} dx.$

(A) $\int_1^2 (x^5 - 4x + 6) dx$

(B) $\int_1^2 (x^5 - 4x^3 + 6) dx$

(C) $\int_1^2 (x^5 - 4x + 6x^{-2}) dx$

(D) $\int_1^2 (x^7 - 4x^3 + 6x^{-2}) dx$

8) Which of the following is equivalent to $\int (2x^3 + 1)^2 dx$?

$u = 2x^3 + 1$

$\frac{du}{dx} = 6x^2$

$= \int u^2 \cdot \frac{1}{6x^2} du$

u-sub does <u>NOT</u> work!

(A) $\int (4x^6 + 4x^3 + 1) dx$

So... just "FOIL" it out and it can be done with power rule!

(B) $\int (2x^3 + 1) dx \cdot \int (2x^3 + 1) dx$

(C) $\int u^2 du$, where $u = 2x^3 + 1$

(D) $\frac{1}{6x^2} \int u^2 du$, where $u = 2x^3 + 1$

<u>cannot</u> move variable expressions out front of an integral

9) Which of the following is equivalent to $\int \cos(4x)\sin^5(4x)dx$?

ALWAYS LET
U = higher powered trig function

$= \int \cancel{\cos(4x)} \cdot u^5 \cdot \frac{1}{4\cancel{\cos 4x}} du$

$U = (\sin 4x)$

$\frac{du}{dx} = 4\cos 4x$

$= \int \frac{1}{4} u^5 du$

Ⓐ $\frac{1}{4}\int \cos u du \cdot \frac{1}{4}\int \sin^5 u du$, where $u = 4x$

CANNOT SPLIT UP PRODUCTS

Ⓑ $\int \cos(4x)dx \cdot \int \sin^5(4x)dx$

Ⓒ $\frac{1}{4}\int u^5 du$, where $u = \sin(4x)$

Ⓓ $\int u^5 du$, where $u = \sin(4x)$

10) Which of the following is equivalent to $\int_1^{e^6} \frac{1}{x(2+\ln x)}dx$?

$U = 2 + \ln x$

$\frac{du}{dx} = \frac{1}{x}$

$= \int_2^8 \frac{1}{\cancel{x} \cdot u} \cdot \cancel{x} du$

$\int_2^8 \frac{1}{u} du$

Ⓐ $\int_2^8 \frac{1}{u}du$

Ⓑ $\int_1^{e^6} \frac{1}{u}du$

Ⓒ $\frac{1}{\int_1^{e^6} x(2+\ln x)dx}$

Ⓓ $\int_1^{e^6} \frac{1}{x}dx \cdot \int_1^{e^6} \frac{1}{2+\ln x}dx$

11) $\int e^x \sin(6e^x+3)dx =$

(A) $-e^x\cos(6e^x+3)+C$

(B) $-6\cos(6e^x+3)+C$

(C) $-\frac{1}{6}\cos(6e^x+3)+C$

(D) $\frac{1}{6}\cos(6e^x+3)+C$

$u = 6e^x + 3$

$\frac{du}{dx} = 6e^x$

$= \int \cancel{e^x} \cdot \sin u \cdot \frac{1}{6\cancel{e^x}}\, du$

$= \frac{1}{6}\int \sin u\, du$

$= -\frac{1}{6}\cos u + C$

$-\frac{1}{6}\cos(6e^x+3) + C$

12) What are all solutions to the differential equation $\frac{dy}{dx} = \frac{1}{x(\ln 2)}$?

(A) $y = \frac{\ln x}{\ln 2} + C$

(B) $y = \frac{x^{(1-\ln 2)}}{(1-\ln 2)} + C$

(C) $y = \frac{-1}{x^2(\ln 2)} + C$

(D) $y = e^{x(\ln 2)} + C$

$1\, dy = \frac{1}{\ln 2} \cdot \frac{1}{x}\, dx$

$\int 1\, dy = \frac{1}{\ln 2}\int \frac{1}{x}\, dx$

$y = \frac{1}{\ln 2} \cdot \ln|x| + C$

13) What is the general solution to the differential equation $\frac{dy}{dx} = \frac{\cos x e^{\sin x}}{\cos y}$?

(A) $y = \arcsin(e^{\sin x}) + C$

(B) $y = \arcsin(e^{\sin x} + C)$

(C) $y = \sin x + \arcsin(C)$

(D) $y = \arcsin(\sin x e^{-\cos x} + C)$

$\cos y \, dy = \cos x \, e^{\sin x} dx$

$\int \cos y \, dy = \int \cos x \, e^{\sin x} dx$

$\sin y = \int \cancel{\cos x} \cdot e^{u} \cdot \frac{1}{\cancel{\cos x}} du$

$u = \sin x$

$\frac{du}{dx} = \cos x$

$\sin y = \int e^{u} du$

$\sin y = e^{u} + C$

$\sin y = e^{\sin x} + C$

$y = \sin^{-1}(e^{\sin x} + c)$

14) What is the general solution to the differential equation $\frac{dy}{dx} = \frac{x\cos(x^2)}{4y}$ for $y > 0$?

(A) $y = \frac{1}{2}\sqrt{\sin(x^2)} + C$

(B) $y = \sqrt{\frac{1}{4}\sin(x^2) + C}$

(C) $y = \frac{1}{8}\sin(x^2) + C$

(D) $y = Ce^{\frac{1}{8}\sin(x^2)}$

$4y \, dy = x\cos(x^2) dx$

$\int 4y \, dy = \int x\cos(x^2) dx$

$2y^2 = \int \cancel{x} \cdot \cos(u) \cdot \frac{1}{2\cancel{x}} du$

$2y^2 = \frac{1}{2}\int \cos u \, du$

$u = x^2$

$\frac{du}{dx} = 2x$

$2y^2 = \frac{1}{2}\sin u + C$

$2y^2 = \frac{1}{2}\sin(x^2) + C$

$y^2 = \frac{1}{4}\sin(x^2) + C$

$y = \cancel{\pm} \sqrt{\frac{1}{4}\sin(x^2) + C}$

15) Which of the following is the solution to the differential equation $\frac{dy}{dx} = (x-2)(y-2)$ for $y > 2$ with the initial condition $y(4) = 5$?

Ⓐ $y = 4 + e^{\left(\frac{x^2}{2} - 2x\right)}$

Ⓑ $y = 2 + 3e^{\left(\frac{x^2}{2} - 2x\right)}$

Ⓒ $y = \frac{4 + \sqrt{16 + 4(x^2 - 4x + 5)}}{2}$

Ⓓ $y = \frac{-x^2 + 4x + 5}{\left(1 - \frac{x^2}{2} + 2x\right)}$

$\frac{1}{y-2}\, dy = x - 2\, dx$

$\int \frac{1}{y-2}\, dy = \int x - 2\, dx$

$u = y - 2$

$\frac{du}{dy} = 1$

$\int \frac{1}{u}\, du = \frac{1}{2}x^2 - 2x + C$

$\ln|u| = \frac{1}{2}x^2 - 2x + C$

solution

$(4,5) \rightarrow$

$\ln|y-2| = \frac{1}{2}x^2 - 2x + C$

$\ln|5-2| = \frac{1}{2}\cdot 16 - 8 + C$

$\ln 3 = 8 - 8 \; C$

$\ln 3 = C$

$\ln|y-2| = \frac{1}{2}x^2 - 2x + \ln 3$

$e^{1/2 x^2 - 2x + \ln 3} = |y-2|$

$\pm e^{1/2 x^2 - 2x + \ln 3} = y - 2$

$2 \pm e^{1/2 x^2 - 2x + \ln 3} = y$

$2 \pm \underbrace{e^{1/2 x^2 - 2x} \cdot e^{\ln 3}}_{\text{by property of exponents}} = y$

$2 \pm 3e^{1/2 x^2 - 2x} = y$

The one that makes $(4,5)$ TRUE

16) $\int \frac{1}{\sqrt{9 - x^2}} dx =$

$u = x$

$\frac{du}{dx} = 1$

$a = 3$

$= \int \frac{1}{\sqrt{a^2 - u^2}}\, du$

$= \sin^{-1}\frac{u}{a} + C$

$= \sin^{-1}\left(\frac{x}{3}\right) + C$

Ⓐ $\ln\left(\sqrt{9 - x^2}\right) + C$

Ⓑ $\frac{1}{3}\sin^{-1}\left(\frac{x}{3}\right) + C$

Ⓒ $3\sin^{-1}\left(\frac{x}{3}\right) + C$

Ⓓ $\sin^{-1}\left(\frac{x}{3}\right) + C$

17) $\int \frac{4}{x^2+4x+8}dx =$

Ⓐ $4\ln|x^2+4x+8|+C$

Ⓑ $\tan^{-1}\left(\frac{x+2}{2}\right)+C$

Ⓒ $4\tan^{-1}(x+2)+C$

Ⓓ $2\tan^{-1}\left(\frac{x+2}{2}\right)+C$

$x^2+4x+\underline{4}+8-\underline{4}$

$(x+2)^2+4$

$= 4\int \frac{1}{(x+2)^2+4}dx$

$u = x+2$

$\frac{du}{dx} = 1$

$a = 2$

$= 4\int \frac{1}{u^2+a^2}du$

$= 4 \cdot \frac{1}{a}\text{Arctan}\,\frac{u}{a}+C$

$= 4 \cdot \frac{1}{2}\text{Arctan}\left(\frac{x+2}{2}\right)+C$

$= 2\tan^{-1}\left(\frac{x+2}{2}\right)+C$

18) $\int \frac{8}{\sqrt{12-x^2-4x}}dx =$

Ⓐ $16\sqrt{12-x^2-4x}+C$

Ⓑ $2\sin^{-1}\left(\frac{x+2}{4}\right)+C$

Ⓒ $8\sin^{-1}\left(\frac{x-2}{4}\right)+C$

Ⓓ $8\sin^{-1}\left(\frac{x+2}{4}\right)+C$

$-x^2-4x+12$

$-(x^2+4x) \quad +12$

$-(x^2+4x+4) \quad +12+4$

$-(x+2)^2+16$

$8\int \frac{1}{\sqrt{16-(x+2)^2}}dx$

$= 8\int \frac{1}{\sqrt{a^2-u^2}}du$

$a = 4$

$u = x+2$

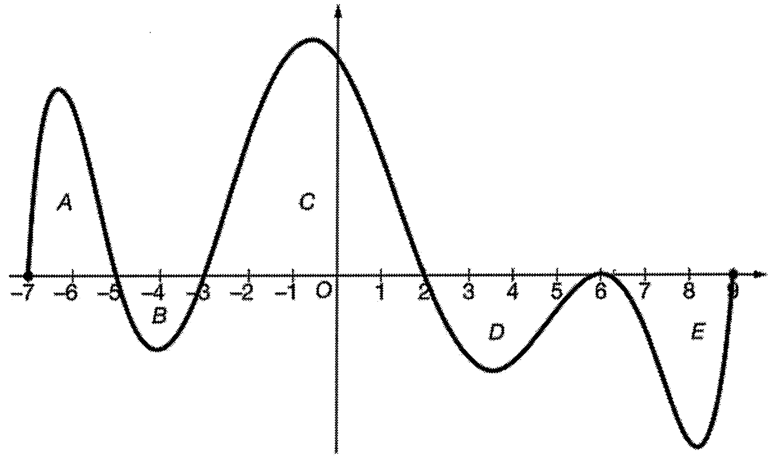
$\frac{du}{dx} = 1$

$= 8\,\text{Arcsin}\,\frac{u}{a}+C$

$= 8\,\text{Arcsin}\left(\frac{x+2}{4}\right)+C$

$= 8\sin^{-1}\left(\frac{x+2}{4}\right)+C$

19) Open Ended



Graph of f

The figure above shows the graph of the continuous function f. The regions $A, B, C, D,$ and E have areas 5, 2, 16, 5, and 6, respectively.

Find the value of $\int_{-7}^{-5} f(2x+16)dx$.

NEEDS u-substitution

$u = 2x+16$

$\frac{du}{dx} = 2$

$= \int_{2}^{6} f(u) \cdot \frac{1}{2} du$

$= \frac{1}{2} \boxed{\int_{2}^{6} f(u) du}$ ← use graph to evaluate

$= \frac{1}{2} \cdot (-5)$

$= \boxed{-\frac{5}{2}}$