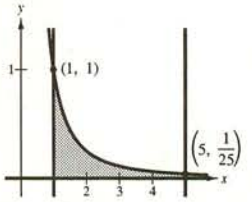
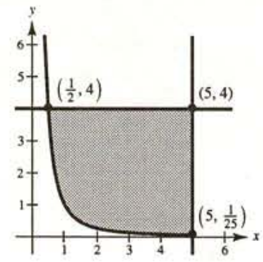
Review Exercises for Chapter 6

1. $A = \int_1^5 \frac{1}{x^2}\,dx = \left[-\frac{1}{x}\right]_1^5 = \frac{4}{5}$



2. $A = \int_{1/2}^5 \left(4 - \frac{1}{x^2}\right) dx$

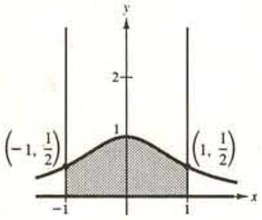
$= \left[4x + \frac{1}{x}\right]_{1/2}^5 = \frac{81}{5}$



3. $A = \int_{-1}^1 \frac{1}{x^2+1}\,dx$

$= \left[\arctan x\right]_{-1}^1$

$= \frac{\pi}{4} - \left(-\frac{\pi}{4}\right) = \frac{\pi}{2}$

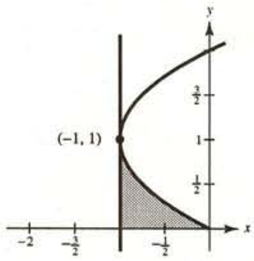


4. $A = \int_0^1 [(y^2 - 2y) - (-1)]\,dy$

$= \int_0^1 (y^2 - 2y + 1)\,dy$

$= \int_0^1 (y - 1)^2\,dy$

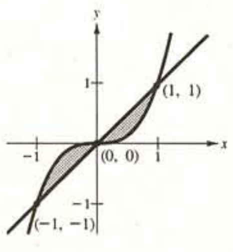
$= \left[\frac{(y-1)^3}{3}\right]_0^1 = \frac{1}{3}$



5. $A = 2\int_0^1 (x - x^3)\,dx$

$= 2\left[\frac{1}{2}x^2 - \frac{1}{4}x^4\right]_0^1$

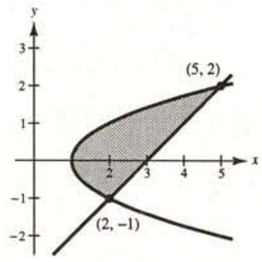
$= \frac{1}{2}$



6. $A = \int_{-1}^2 [(y + 3) - (y^2 + 1)]\,dy$

$= \int_{-1}^2 (2 + y - y^2)\,dy$

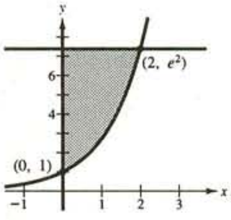
$= \left[2y + \frac{1}{2}y^2 - \frac{1}{3}y^3\right]_{-1}^2 = \frac{9}{2}$



7. $A = \int_0^2 (e^2 - e^x)\,dx$

$= \left[xe^2 - e^x\right]_0^2$

$= e^2 + 1$

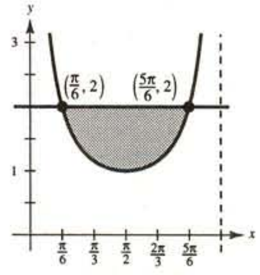


8. $A = 2\int_{\pi/6}^{\pi/2} (2 - \csc x)\,dx$

$= 2\left[2x - \ln|\csc x - \cot x|\right]_{\pi/6}^{\pi/2}$

$= 2\left([\pi - 0] - \left[\frac{\pi}{3} - \ln(2 - \sqrt{3})\right]\right)$

$= 2\left[\frac{2\pi}{3} + \ln(2 - \sqrt{3})\right] \approx 1.555$

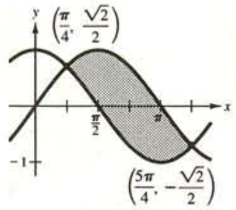


9. $A = \int_{\pi/4}^{5\pi/4} (\sin x - \cos x)\,dx$

$= \left[-\cos x - \sin x\right]_{\pi/4}^{5\pi/4}$

$= \left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right) - \left(-\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}\right)$

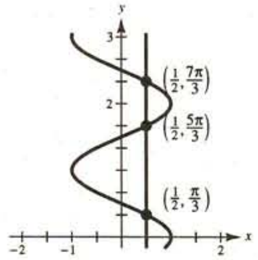
$= \frac{4}{\sqrt{2}} = 2\sqrt{2}$



10. $A = \int_{\pi/3}^{5\pi/3} \left(\frac{1}{2} - \cos y\right) dy + \int_{5\pi/3}^{7\pi/3} \left(\cos y - \frac{1}{2}\right) dy$

$= \left[\frac{y}{2} - \sin y\right]_{\pi/3}^{5\pi/3} + \left[\sin y - \frac{y}{2}\right]_{5\pi/3}^{7\pi/3}$

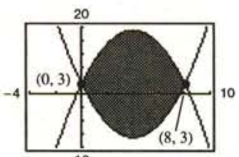
$= \frac{\pi}{3} + 2\sqrt{3}$



11. $A = \int_0^8 [(3 + 8x - x^2) - (x^2 - 8x + 3)]\,dx$

$= \int_0^8 (16x - 2x^2)\,dx$

$= \left[8x^2 - \frac{2}{3}x^3\right]_0^8 = \frac{512}{3} \approx 170.667$



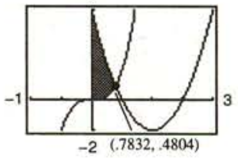
12. Point of intersection is given by:

$x^3 - x^2 + 4x - 3 = 0 \Rightarrow x \approx 0.783.$

$A \approx \int_0^{0.783} (3 - 4x + x^2 - x^3)\,dx$

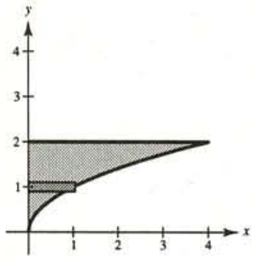
$= \left[3x - 2x^2 + \frac{1}{3}x^3 - \frac{1}{4}x^4\right]_0^{0.783}$

≈ 1.189



22. (a) **Shell**

$$V = 2\pi \int_0^2 y^3\,dy = \left[\frac{\pi}{2}y^4\right]_0^2 = 8\pi$$

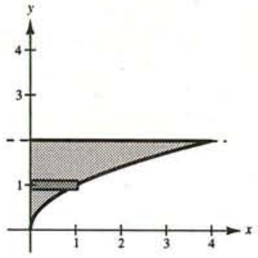


(b) **Shell**

$$V = 2\pi \int_0^2 (2 - y)y^2\,dy$$

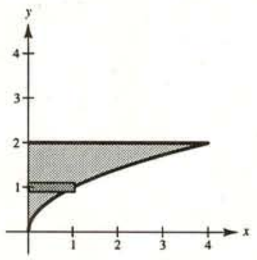
$$= 2\pi \int_0^2 (2y^2 - y^3)\,dy$$

$$= 2\pi\left[\frac{2}{3}y^3 - \frac{1}{4}y^4\right]_0^2 = \frac{8\pi}{3}$$



(c) **Disc**

$$V = \pi \int_0^2 y^4\,dy = \left[\frac{\pi}{5}y^5\right]_0^2 = \frac{32\pi}{5}$$

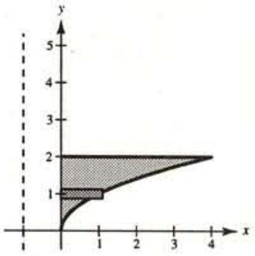


(d) **Disc**

$$V = \pi \int_0^2 [(y^2 + 1)^2 - 1^2]\,dy$$

$$= \pi \int_0^2 (y^4 + 2y^2)\,dy$$

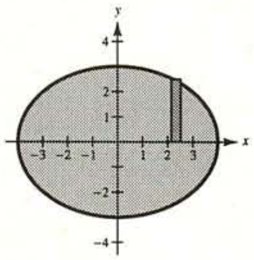
$$= \pi\left[\frac{1}{5}y^5 + \frac{2}{3}y^3\right]_0^2 = \frac{176\pi}{15}$$



23. (a) **Shell**

$$V = 4\pi \int_0^4 x\left(\frac{3}{4}\right)\sqrt{16 - x^2}\,dx$$

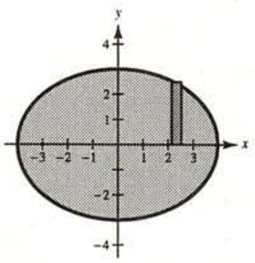
$$= \left[3\pi\left(-\frac{1}{2}\right)\left(\frac{2}{3}\right)(16 - x^2)^{3/2}\right]_0^4 = 64\pi$$



(b) **Disc**

$$V = 2\pi \int_0^4 \left[\frac{3}{4}\sqrt{16 - x^2}\right]^2 dx$$

$$= \frac{9\pi}{8}\left[16x - \frac{x^3}{3}\right]_0^4 = 48\pi$$

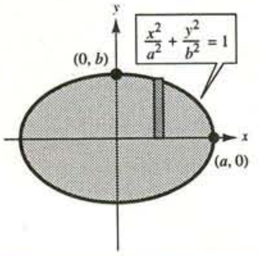


24. (a) **Shell**

$$V = 4\pi \int_0^a (x)\frac{b}{a}\sqrt{a^2 - x^2}\,dx$$

$$= \frac{-2\pi b}{a}\int_0^a (a^2 - x^2)^{1/2}(-2x)\,dx$$

$$= \left[\frac{-4\pi b}{3a}(a^2 - x^2)^{3/2}\right]_0^a = \frac{4}{3}\pi a^2 b$$

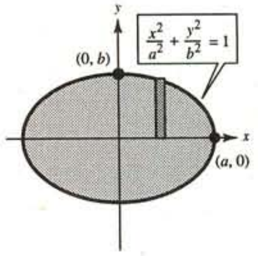


(b) **Disc**

$$V = 2\pi \int_0^a \frac{b^2}{a^2}(a^2 - x^2)\,dx$$

$$= \frac{2\pi b^2}{a^2}\left[a^2 x - \frac{1}{3}x^3\right]_0^a$$

$$= \frac{4}{3}\pi a b^2$$



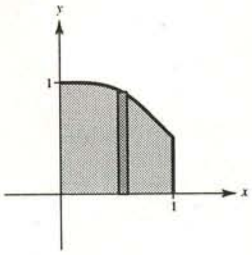
25. Shell

$$V = 2\pi \int_0^1 \frac{x}{x^4 + 1}\,dx$$

$$= \pi \int_0^1 \frac{(2x)}{(x^2)^2 + 1}\,dx$$

$$= \left[\pi \arctan(x^2)\right]_0^1$$

$$= \pi\left[\frac{\pi}{4} - 0\right] = \frac{\pi^2}{4}$$



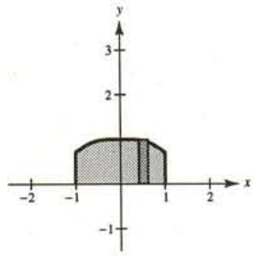
26. Disc

$$V = 2\pi \int_0^1 \left[\frac{1}{\sqrt{1 + x^2}}\right]^2 dx$$

$$= \left[2\pi \arctan x\right]_0^1$$

$$= 2\pi\left(\frac{\pi}{4} - 0\right)$$

$$= \frac{\pi^2}{2}$$



27. Shell

$u = \sqrt{x - 2}$

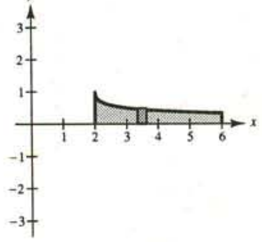
$x = u^2 + 2$

$dx = 2u\,du$

$$V = 2\pi \int_2^6 \frac{x}{1 + \sqrt{x - 2}}\,dx = 4\pi \int_0^2 \frac{(u^2 + 2)u}{1 + u}\,du$$

$$= 4\pi \int_0^2 \frac{u^3 + 2u}{1 + u}\,du = 4\pi \int_0^2 \left(u^2 - u + 3 - \frac{3}{1 + u}\right)du$$

$$= 4\pi\left[\frac{1}{3}u^3 - \frac{1}{2}u^2 + 3u - 3\ln(1 + u)\right]_0^2 = \frac{4\pi}{3}(20 - 9\ln 3) \approx 42.359$$

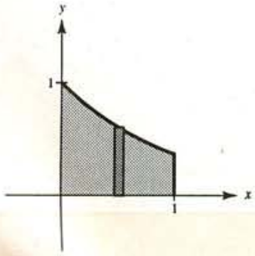


28. Disc

$$V = \pi \int_0^1 (e^{-x})^2\, dx$$

$$= \pi \int_0^1 e^{-2x}\, dx = \left[-\frac{\pi}{2}e^{-2x}\right]_0^1$$

$$= \left(\frac{-\pi}{2e^2} + \frac{\pi}{2}\right) = \frac{\pi}{2}\left(1 - \frac{1}{e^2}\right)$$



29. Since $y \le 0, A = -\int_{-1}^{0} x\sqrt{x+1}\, dx.$

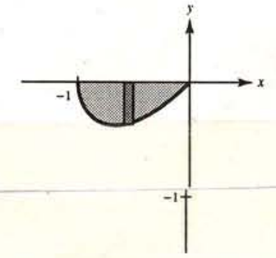
$$u = x + 1$$

$$x = u - 1$$

$$dx = du$$

$$A = -\int_0^1 (u-1)\sqrt{u}\, du = -\int_0^1 (u^{3/2} - u^{1/2})\, du$$

$$= -\left[\frac{2}{5}u^{5/2} - \frac{2}{3}u^{3/2}\right]_0^1 = \frac{4}{15}$$

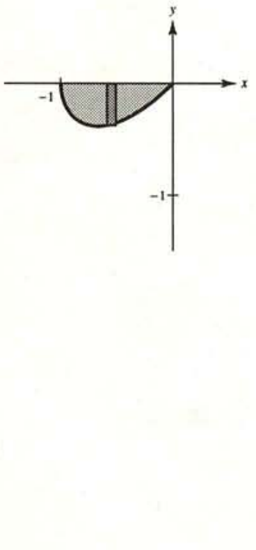


30. (a) **Disc**

$$V = \pi \int_{-1}^{0} x^2(x+1)\, dx$$

$$= \pi \int_{-1}^{0} (x^3 + x^2)\, dx$$

$$= \pi\left[\frac{x^4}{4} + \frac{x^3}{3}\right]_{-1}^{0} = \frac{\pi}{12}$$



(b) **Shell**

$$u = \sqrt{x+1}$$

$$x = u^2 - 1$$

$$dx = 2u\, du$$

$$V = 2\pi \int_{-1}^{0} x^2\sqrt{x+1}\, dx$$

$$= 4\pi \int_0^1 (u^2 - 1)^2 u^2 du$$

$$= 4\pi \int_0^1 (u^6 - 2u^4 + u^2)\, du$$

$$= 4\pi\left[\frac{1}{7}u^7 - \frac{2}{5}u^5 + \frac{1}{3}u^3\right]_0^1 = \frac{32\pi}{105}$$