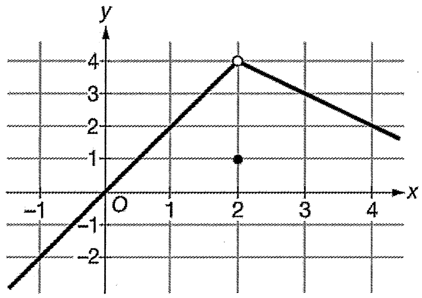
Unit 1 AP Classroom Practice for Sections 1-2

1)



Graph of f

What is the $\lim\limits_{x \to 2} f(x)$?

(A) $\frac{1}{2}$

"what it approaches" NOT necessarily what it equals!

(B) 1

(C) 4

(D) The limit does not exist.

2) Of the following tables, which best reflects the values of a function g for which $\lim\limits_{x \to 7} g(x) = 6$?

(A)

x	5.85	5.90	5.95	5.99	6.01	6.05	6.10	6.15
$g(x)$	7.126	7.075	7.033	7.006	6.995	6.977	6.964	6.960

7

(B)

x	6.85	6.90	6.95	6.99	7.01	7.05	7.10	7.15
$g(x)$	5.620	5.837	5.961	5.998	5.999	5.964	5.863	5.709

closer to 6 → ← closer to 6

(C)

x	6.85	6.90	6.95	6.99	7.01	7.05	7.10	7.15
$g(x)$	5.919	5.942	5.969	5.993	7.017	7.087	7.177	7.269

(D)

x	6.85	6.90	6.95	6.99	7.01	7.05	7.10	7.15
$g(x)$	1.362	5.954	10.691	14.690	6.010	6.049	6.095	6.140

3) Let f be a function that is defined for all real numbers x. Of the following, which is the best interpretation of the statement $\lim_{x \to 3} f(x) = 5$?

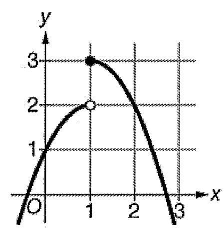
(A) The value of the function f at $x = 3$ is 5. NOT NECISSARILY

(B) The value of the function f at $x = 5$ is 3.

(C) As x approaches 3, the values of $f(x)$ approach 5.

(D) As x approaches 5, the values of $f(x)$ approach 3.

4)



Graph of f

The graph of a function f is shown above. Which of the following statements is true?

(A) $\lim_{x \to 1} f(x) = 2.5$

(B) $\lim_{x \to 1} f(x) = 3$

(C) $\lim_{x \to 1} f(x)$ does not exist because the left-hand and right-hand limits of $f(x)$ as x approaches 1 do not exist.

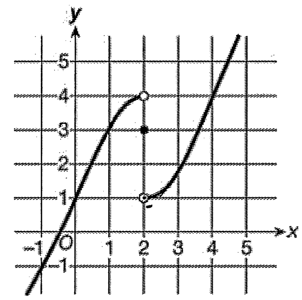
yes they do

$\lim_{x \to 1^+} f(x) = 3$

$\lim_{x \to 1^-} f(x) = 2$

(D) $\lim_{x \to 1} f(x)$ does not exist because while the left-hand and right-hand limits of $f(x)$ as x approaches 1 exist, their values are not equal.

5)



Graph of f

The graph of the function f is shown above. What is $\lim_{x \to 2^+} f(x)$?

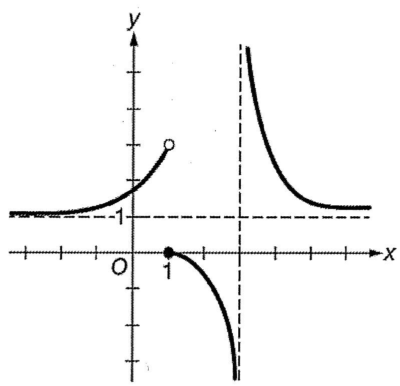
Ⓐ 1

Ⓑ 3

Ⓒ 4

Ⓓ nonexistent

6)



Graph of f

The graph of a function f is shown above. Which of the following statements is true?

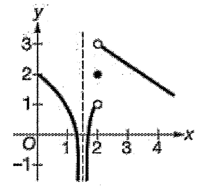
Ⓐ $\lim_{x \to 1} f(x) = 1.5$

Ⓑ $\lim_{x \to 1} f(x) = 0$

Ⓒ $\lim_{x \to 1} f(x)$ does not exist because the left-hand and right-hand limits of $f(x)$ as x approaches 1 do not exist.

Ⓓ $\lim_{x \to 1} f(x)$ does not exist because while the left-hand and right-hand limits of $f(x)$ as x approaches 1 exist, their values are not equal.

7)



Graph of f

The graph of the function f is shown above. What is $\lim_{x \to 2^+} f(x)$?

(A) 3

(B) 2

(C) 1

(D) nonexistent

4

8)

x	3.9	3.99	3.999	3.9999	4.0001	4.001	4.01	4.1
$f(x)$	5.8	5.85	5.9	5.95	6.999	6.99	6.9	6

FROM LEFT → ← FROM RIGHT

The table above gives values of the function f at selected values of x. Which of the following conclusions is supported by the data in the table?

(A) $\lim_{x \to 4} f(x) = 6$

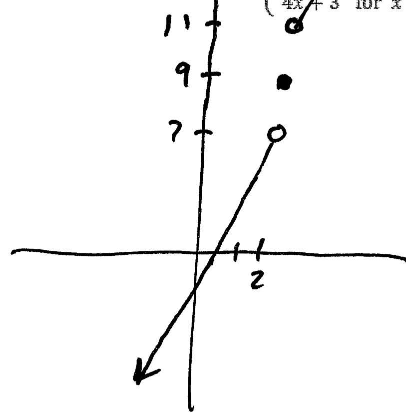
(B) $\lim_{x \to 4} f(x) = 7$

(C) $\lim_{x \to 4^-} f(x) = 6$ and $\lim_{x \to 4^+} f(x) = 7$

(D) $\lim_{x \to 4^-} f(x) = 7$ and $\lim_{x \to 4^+} f(x) = 6$

9)

$$f(x) = \begin{cases} 5x - 3 & \text{for } x < 2 \\ 9 & \text{for } x = 2 \\ 4x + 3 & \text{for } x > 2 \end{cases}$$



Let f be the piecewise function defined above. The value of $\lim_{x \to 2^+} f(x)$ is

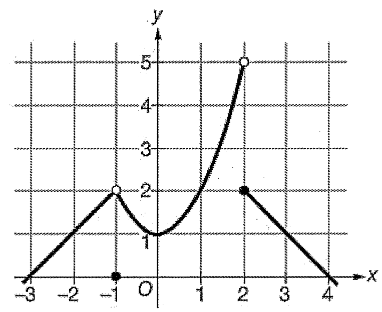
(A) 7

(B) 9

(C) 11

(D) nonexistent

10)



Graph of f

The graph of the function f is shown above. What is $\lim_{x \to -1} f(f(x))$?

(A) 1

(B) 2

(C) 5

(D) nonexistent

$= f\left(\lim_{x \to -1} f(x)\right)$

$= f(2)$

$= 2$

11)

$f(2) = 3$	$\lim_{x \to 2} f(x) = 4$
$g(2) = -6$	$\lim_{x \to 2} g(x) = -6$
$h(2) = -3$	$\lim_{x \to 2} h(x) = 2$

The table above gives selected values and limits of the functions f, g, and h. What is $\lim_{x \to 2} (h(x)(5f(x) + g(x)))$?

(A) -27

(B) -20

(C) 28

(D) 34

$= \lim_{x \to 2} h(x) \cdot \left(5 \lim_{x \to 2} f(x) + \lim_{x \to 2} g(x)\right)$

$= \lim_{x \to 2} h(x) \cdot \left(5 \lim_{x \to 2} f(x) + \lim_{x \to 2} g(x)\right)$

$= (2) \cdot \left(5(4) + (-6)\right)$

$2\left(20 - 6\right)$

12) If f is the function defined by $f(x) = \frac{x^2-4}{x^2+x-6}$, then $\lim\limits_{x \to 2} f(x)$ is

"maybe nonexistent"

$$\lim_{x \to 2} f(x) = \frac{0}{0}$$

$$= \lim_{x \to 2} \frac{(x+2)\cancel{(x-2)}}{(x+3)\cancel{(x-2)}}$$

NOT nonexistent as the zero in denominator can be "simplified out".

$$= \lim_{x \to 2} \frac{x+2}{x+3}$$

$$= \frac{4}{5}$$

(A) 0

(B) $\frac{2}{3}$

(C) $\frac{4}{5}$

(D) nonexistent

13) If f is the function defined by $f(x) = \frac{x^2-4}{\sqrt{x}-\sqrt{2}}$, then $\lim\limits_{x \to 2} f(x)$ is equivalent to which of the following?

"maybe nonexistent"

$$\lim_{x \to 2} f(x) = \frac{0}{0}$$

$$= \lim_{x \to 2} \frac{x^2-4}{\sqrt{x}-\sqrt{2}} \cdot \frac{\sqrt{x}+\sqrt{2}}{\sqrt{x}+\sqrt{2}}$$

$$= \lim_{x \to 2} \frac{(x+2)\cancel{(x-2)}(\sqrt{x}+\sqrt{2})}{\cancel{(x-2)}}$$

$$= \lim_{x \to 2} (x+2)(\sqrt{x}+\sqrt{2})$$

(A) $\lim\limits_{x \to 2} (x+2)\left(\sqrt{x}+\sqrt{2}\right)$

(B) $\lim\limits_{x \to 2} \left(\sqrt{x}+\sqrt{2}\right)$

(C) $\lim\limits_{x \to 2} \left(x\sqrt{x}-2\sqrt{2}\right)$

(D) $\lim\limits_{x \to 2} (x+2)\left(x^2+4\right)$

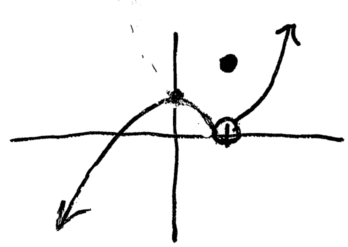
14)

$$f(x) = \begin{cases} \frac{(x-1)^2(x+1)}{|x-1|} & \text{for } x \neq 1 \\ 2 & \text{for } x = 1 \end{cases}$$

$$\frac{(x-1)(x-1)(x+1)}{|x-1|}$$

$$\frac{x-1}{|x-1|} \cdot (x^2-1)$$

$$\begin{cases} +1 \cdot (x^2-1) & x > 1 \\ -1 \cdot (x^2-1) & x < 1 \\ 2 & x = 1 \end{cases}$$



If f is the function defined above, then $\lim\limits_{x \to 1} f(x)$ is

(A) 0

(B) 1

(C) 2

(D) nonexistent

15) If h is a piecewise linear function such that $\lim\limits_{x \to 3} h(x)$ does not exist, which of the following could be representative of the function h?

$\lim\limits_{x \to 3^+} h(x) = 9$

$\lim\limits_{x \to 3^-} h(x) = 9$

I. $h(x) = \begin{cases} 12 - x & \text{for } x < 3 \\ 4x - 3 & \text{for } x > 3 \end{cases}$

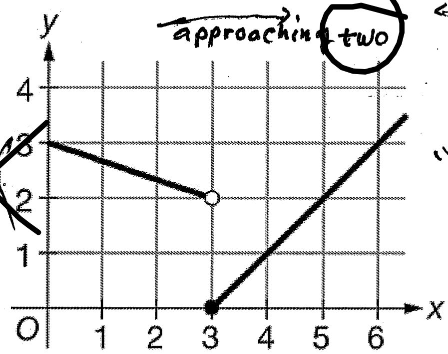
II.

x	0	1	2	3	4	5	6
$h(x)$	3	$\frac{8}{3}$	$\frac{7}{3}$	0	1	2	3

approaching two

approaching zero

III.



"NOT same from left & right"

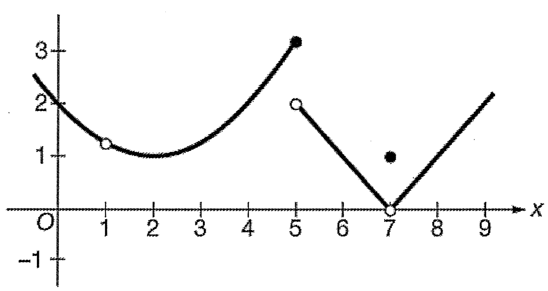
(A) I only

(B) II only

(C) III only

(D) II and III only

16)



Graph of f

The graph of the function f is shown above. What are all values of x for which f has a removable discontinuity?

only "one" point needed to make continuous

either make one or move one

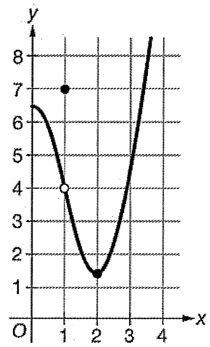
(A) 1 only

(B) 5 only

(C) 1 and 7 only

(D) 1, 5, and 7

17)



Graph of f

The graph of the function f is shown above. Which of the following could be a table of values for f?

(A)

x	0.95	0.99	1	1.001	1.05
$f(x)$	6.9025	6.9801	7	7.0020	7.1025

(B)

x	0.95	0.99	1	1.001	1.05
$f(x)$	4.1624	4.0325	7	3.9968	3.8376

(C)

x	0.95	0.99	1	1.001	1.05
$f(x)$	4.1624	4.0325	4	3.9968	3.8376

(D)

x	0.95	0.99	1	1.001	1.05
$f(x)$	4.1624	4.0325	undefined	3.9968	3.8376

18) Let f be the function defined by $f(x) = \frac{3x^3+2x^2}{x^2-x}$. Which of the following statements is true?

(A) f has a discontinuity due to a vertical asymptote at $x = 0$ and at $x = 1$.

(B) f has a removable discontinuity at $x = 0$ and a jump discontinuity at $x = 1$.

(C) f has a removable discontinuity at $x = 0$ and a discontinuity due to a vertical asymptote at $x = 1$.

(D) f is continuous at $x = 0$, and f has a discontinuity due to a vertical asymptote at $x = 1$.

hole in graph at $x=0$ since "x" in denom CAN be simplified out!

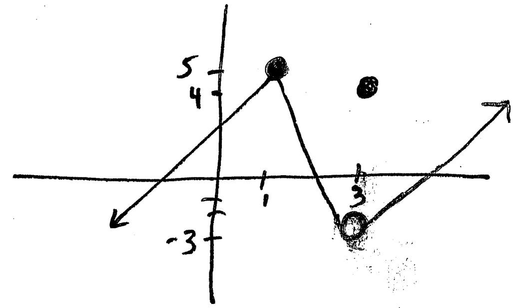
$f(x) = \frac{\cancel{x}^2(3x+2)}{\cancel{x}(x-1)}$

$f(x) = \frac{x(3x+2)}{(x-1)}$

V.A. at $x=1$ since this factor cannot be removed

19)

$$f(x)=\begin{cases}2x+3 & \text{for } x<1\\ 5 & \text{for } x=1\\ -4x+9 & \text{for } 1<x<3\\ 4 & \text{for } x=3\\ x-6 & \text{for } x>3\end{cases}$$



Let f be the piecewise function defined above. Which of the following statements is false?

(A) f is continuous at $x=1$.

(B) f is continuous at $x=2$.

(C) f is continuous at $x=3$.

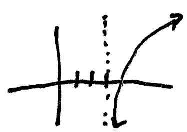
(D) f is continuous at $x=4$.

20) Which of the following functions are continuous on the interval $0<x<5$?

I. $f(x)=\frac{x-3}{x^2-9}$ $\quad f(x)=\frac{x-3}{(x+3)(x-3)}$ $\quad f(x)=\frac{1}{x+3}$ with hole in graph at $x=3$ so No!

II. $g(x)=\frac{x-3}{x^2+9}$ ← Always defined so yes!

III. $h(x)=\ln(x-3)$



NOT defined $0<x<3$ so NO!

(A) II only

(B) I and II only

(C) I and III only

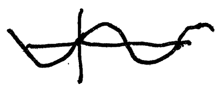
(D) II and III only

21) Which of the following functions is not continuous on the interval $-\infty < x < \infty$?

(A) $f(x) = x^4 + x^3 + x^2 + x + 1$ polynomials always continuous

(B) $g(x) = \frac{1}{(x^3+x^2)+(x+1)}$ $g(x) = \frac{1}{(x^2+1)(x+1)}$ so. vertical asymptote at $x = -1$

factor by grouping ↑

(C) $h(x) = \frac{\pi}{2}\sin x$  .. sin function always continuous

(D) $k(x) = \frac{1}{1+e^{-x}}$ $1 + e^{-x}$ NEVER zero so always continuous

22)

$$f(x) = \begin{cases} x^2 + b^2 & \text{for } x < 2 \\ bx + 2b & \text{for } x \geq 2 \end{cases}$$

Let f be the function defined above, where b is a constant. For what values of b, if any, is f continuous at $x = 2$?

$$\lim_{x \to 2^-} f(x) = \lim_{x \to 2^+} f(x) = f(2)$$

↑ same as $\lim_{x \to 2^+} f(x)$

$$4 + b^2 = 2b + 2b$$

$$4 + b^2 = 4b$$

$$b^2 - 4b + 4 = 0$$

$$(b-2)(b-2) = 0$$

$$b - 2 = 0$$

$$b = 2$$

(A) 0 only

(B) 2 only

(C) 0 and 2

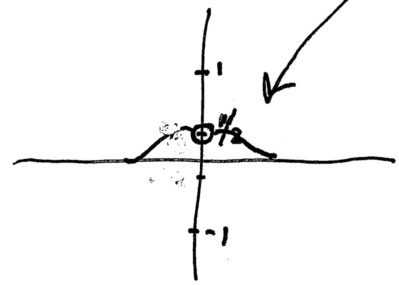
(D) There is no such b.

$$\frac{\sin(3x)}{6x} = \frac{1}{2}\cdot\frac{\sin(3x)}{3x}$$

23)

$$f(x) = \begin{cases} \frac{\sin(3x)}{6x} & \text{for } x \neq 0 \\ c & \text{for } x = 0 \end{cases}$$

The function f is defined above, where c is a constant. For what value of c is f continuous at $x = 0$?



(A) 0

(B) $\frac{1}{2}$

(C) 1

(D) 2

24) **CALCULATOR NEEDED**

$$f(x) = \begin{cases} 2 - \sin x & \text{for } x \leq 1 \\ cx\sqrt{x^2+2}+c & \text{for } x > 1 \end{cases}.$$

Let f be the function defined above, where c is a constant. For what value of c is f continuous for all x ?

$$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^+} f(x) = f(1)$$

same as $\lim_{x\to 1^+} f(x)$

(A) 1.159

(B) 0.424

(C) 0.409

(D) There is no such value of c.

$$2 - \sin(1) = c\sqrt{1+2} + c$$

$$\boxed{2 - \sin(1) = c\cdot\sqrt{3} + c}$$

Solve using calculator and intersect function

$$Y_1 = Y_2$$

25)

$$f(x)=\begin{cases}\sin x & \text{for } x<0\\ \cos x & \text{for } 0\le x\le \frac{3\pi}{2}\\ \tan x & \text{for } \frac{3\pi}{2}<x\le 2\pi\\ \cot x & \text{for } 2\pi<x\le \frac{5\pi}{2}\end{cases}$$

Let f be the function given above. On which of the following intervals is f continuous?

(A) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

(B) $\left(\frac{\pi}{4}, \pi\right)$

(C) $\left(\pi, \frac{7\pi}{4}\right)$

(D) $\left(\frac{7\pi}{4}, \frac{5\pi}{2}\right)$