Unit 1 AP Classroom Practice for Sections 3-5

1) Let g and h be the functions defined by $g(x) = \sin\left(\frac{\pi}{2}x\right) + 4$ and $h(x) = -\frac{1}{4}x^3 + \frac{3}{4}x + \frac{9}{2}$. If f is a function that satisfies $g(x) \le f(x) \le h(x)$ for $-1 < x < 2$, what is $\lim_{x \to 1} f(x)$?

$g(1) = \sin\frac{\pi}{2} + 4$
$= 5$

$h(1) = -\frac{1}{4} + \frac{3}{4} + \frac{9}{2}$
$= 5$

(A) 4

(B) $\frac{9}{2}$

(C) 5

(D) The limit cannot be determined from the information given.

2) CALCULATOR ALLOWED!

Let f, g, and h be the functions defined by $f(x) = \frac{\sin x}{2x}$, $g(x) = x^4 \cos\left(\frac{1}{x^2}\right)$, and $h(x) = \frac{x^2}{\tan x}$ for $x \neq 0$. All of the following inequalities are true on the interval $[-1, 1]$ for $x \neq 0$. Which of the inequalities can be used with the squeeze theorem to find the limit of the function as x approaches 0 ?

$\lim_{x \to 0} f(x) = \frac{1}{2}$ $\quad$ $\lim_{x \to 0} g(x) = 0$ $\quad$ $\lim_{x \to 0} h(x) = 0$

I. $\frac{1}{4} \le f(x) \le x^2 + \frac{1}{2}$

II. $-x^4 \le g(x) \le x^4$

III. $-\frac{1}{x^2} \le h(x) \le \frac{1}{x^2}$

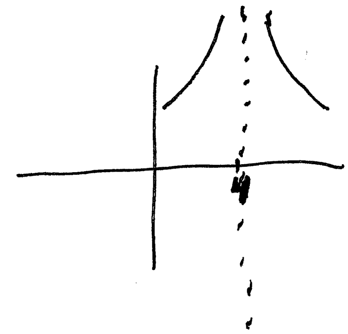
(A) I only

(B) II only $\quad$ only one where all "3" functions have same limit

(C) I and III only

(D) II and III only

3) The function g is continuous at all x except $x = 4$. If $\lim\limits_{x \to 4} g(x) = \infty$, which of the following statements about g must be true?



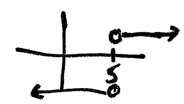
(A) $g(4) = \infty$

(B) The line $x = 4$ is a horizontal asymptote to the graph of g.

(C) The line $x = 4$ is a vertical asymptote to the graph of g.

(D) The line $y = 4$ is a vertical asymptote to the graph of g.

$y = \frac{|x-5|}{x-5}$



NO V.A. ↑

4) The function g is defined by $g(x) = \frac{|x-5|}{x-5}\ln\left(\frac{x+5}{x^2}\right)$. At what values of x does the graph of g have a vertical asymptote?

$y = \ln(\ )$

V.A. when this equals zero

So $x = -5$

(A) $x = -5$ only

(B) $x = 0$ only

(C) $x = -5$ and $x = 0$ only

(D) $x = -5$, $x = 0$, and $x = 5$

$y = \frac{1}{x^2}$

V.A. when this equals zero and cannot be canceled out

So $x = 0$

hole, asymptote or jump

5) The function g is continuous at all x except $x = 2$. If $\lim_{x \to 2^-} g(x) = \infty$, which of the following statements about g must be true?

heading to infinite large velues so must be an asymptote

Ⓐ $g(2) = \infty$ "we never say this"

"$x =$" is a vertical line

Ⓑ The line $x = 2$ is a horizontal asymptote to the graph of g. ?

Ⓒ The line $x = 2$ is a vertical asymptote to the graph of g.

"$y =$" is a horizontal line

Ⓓ The line $y = 2$ is a vertical asymptote to the graph of g. ?

6) Let f be the function defined by $f(x) = \frac{1-5x-2x^3}{3x^3+7}$ for $x > 0$. Which of the following is a horizontal asymptote to the graph of f?

Ⓐ $y = -\frac{2}{3}$

Ⓑ $y = \frac{1}{3}$

Ⓒ $y = \frac{2}{3}$

Ⓓ There is no horizontal asymptote to the graph of f.

7) Let f be the function defined by $f(x) = \frac{2^x+5}{e^x+1}$ for $x > 0$. Which of the following is a horizontal asymptote to the graph of f?

$$\lim_{x \to \infty} \frac{2^x + 5}{(2.718\ldots)^x + 1}$$

these don't matter

denominator gets larger QUICKER

SO ...

$y = 0$

- (A) $y = 0$
- (B) $y = \frac{2}{e}$
- (C) $y = 1$
- (D) There is no horizontal asymptote to the graph of f.

8) Let f be the function defined by $f(x) = \frac{3^x+2}{e^{2x}+1}$ for $x > 0$. Which of the following is a horizontal asymptote to the graph of f?

$$\lim_{x \to \infty} \frac{3^x + 2}{(e^2)^x + 1}$$

$$= \lim_{x \to \infty} \frac{3^x + 2}{(7.389)^x + 1}$$

denominator gets larger QUICKER

so

$y = 0$

- (A) $y = 0$
- (B) $y = \frac{3}{e^2}$
- (C) $y = 1$
- (D) There is no horizontal asymptote to the graph of f.

so both $\lim_{x\to\infty}$ & $\lim_{x\to-\infty}$

NO IMPACT as this oscillates

9) Let f be the function given by $f(x) = \frac{(\cos x)e^{2x}-1}{e^{2x-1}+2}$. What are all horizontal asymptotes to the graph of f?

$$\lim_{x\to\infty} \frac{(\cos x)e^{2x}-1}{e^{2x-1}+2}$$

as x gets large, e^{2x} gets bigger quicker than e^{2x-1} so "∞" (NO H.A.)

- (A) $y = -\frac{1}{2}$ only
- (B) $y = e$ only
- (C) $y = -\frac{1}{2}$ and $y = e$
- (D) The graph of f has no horizontal asymptotes.

Remember $e^{-\infty}$ approaches zero!

$$\lim_{x\to-\infty} \frac{(\cos x)e^{2x}-1}{e^{2x-1}+2} = \frac{\cos x(0)-1}{0+2} = -\frac{1}{2}$$

10)

x	-2	-1	0	1	2	3
$f(x)$	-2	5	2	-4	-1	3

one here one here one here

Selected values of a continuous function f are given in the table above. What is the fewest possible number of zeros of f in the interval $[-2, 3]$?

IVT

- (A) Zero
- (B) One
- (C) Two
- (D) Three

11)

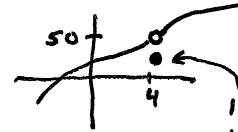
x	0	1	3	7
$g(x)$	24	35	42	68

The table above gives values of a function g at selected values of x. Which of the following statements, if true, would be sufficient to conclude that there exists a number c in the interval $[0, 7]$ such that $g(c) = 50$?

I. g is defined for all x in the interval $[0, 7]$. → this could be possible

II. g is increasing for all x in the interval $[0, 7]$.

III. g is continuous for all x in the interval $[0, 7]$.



it's defined but $g(c)=50$ doesn't happen

(A) II only

(B) III only

(C) I and III only

(D) I, II, and III

12) CALCULATOR ALLOWED!

Let f be the function given by $f(x) = \frac{9+2xe^{-\frac{x}{4}}}{\cos\left(\frac{x}{2}\right)}$. The Intermediate Value Theorem applied to f on the closed interval $[24, 28]$ guarantees a solution in $[24, 28]$ to which of the following equations?

$f(24) \approx 10.806$

$f(28) \approx 66.193$

(A) $f(x) = 0$

(B) $f(x) = 9.090$

(C) $f(x) = 12.235$

(D) $f(x) = 76.999$

13)

x	0	1	2	3	4	5
$f(x)$	1	-5	-4	2	-10	-15

one here one here one here

Selected values of a continuous function f are given in the table above. What is the fewest possible number of zeros of f in the interval $[0, 5]$?

(A) Zero, because $f(x)$ is not equal to 0 for any of the values in the table.

(B) One, because f is continuous on the interval $[0, 5]$ and $f(0) > 0 > f(5)$.

(C) Two, because the values for $f(x)$ in the table change from positive to negative twice.

(D) Three, because f is continuous on the interval $[0, 5]$ and $f(0) > 0 > f(1)$, $f(1) < 0 < f(3)$, and $f(3) > 0 > f(5)$.