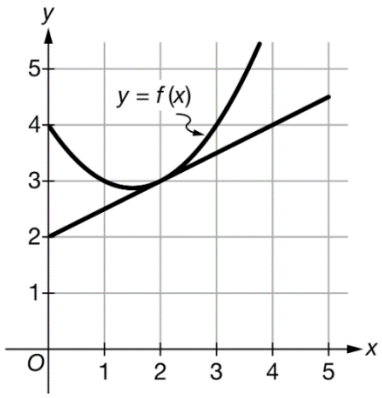
Unit 2 AP Classroom Practice for Sections 1-5

1)



Shown above is the graph of the differentiable function f, along with the line tangent to the graph of f at $x = 2$. What is the value of $f'(2)$?

(A) $\frac{1}{2}$

(B) 2

(C) 3

(D) 4

2) An equation for the line tangent to the graph of the differentiable function f at $x = 3$ is $y = 4x + 6$. Which of the following statements must be true?

I. $f(0) = 6$

II. $f(3) = 18$

III. $f'(3) = 4$

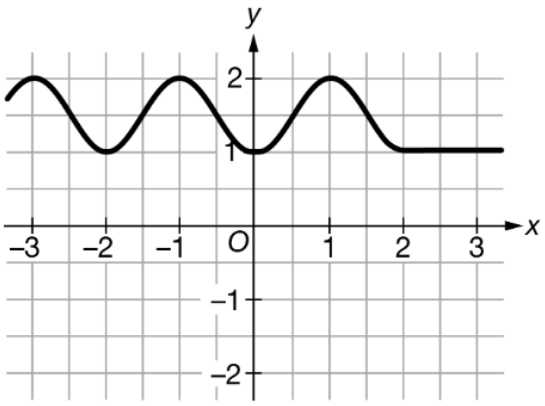
(A) None

(B) I and II only

(C) II and III only

(D) I, II, and III

3)



Graph of f'

Let f be a differentiable function with $f(1) = 3$. The graph of f', the derivative of f, is shown above.

Which of the following statements is true about the line tangent to the graph of f at $x = 1$?

(A) The tangent line has slope 2 and passes through the point $(1, 3)$.

(B) The tangent line has slope 2 and passes through the point $(1, 2)$.

(C) The tangent line has slope 0 and passes through the point $(1, 3)$.

(D) The tangent line has slope 0 and passes through the point $(1, 2)$.

4) GRAPHING CALCULATOR NEEDED

Let f be the function given by $f(x) = x^4 + \frac{1}{2}x^3 - 5x^2 + \tan\left(\frac{x}{2}\right)$. Of the following values of x, at which does the line tangent to the graph of f have the greatest slope?

(A) $x = -2$

(B) $x = -1$

(C) $x = 0$

(D) $x = 1$

5) GRAPHING CALCULATOR NEEDED

Let f be the function given by $f(x) = \cos x - \csc x$. What is the value of $f'(1)$?

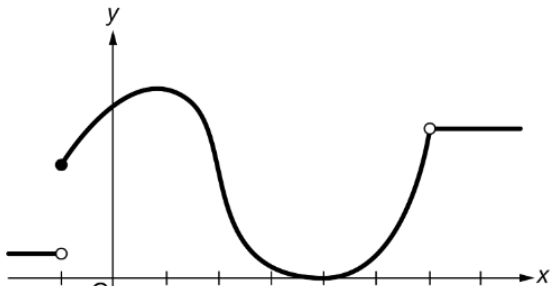
(A) $f'(1)$ is undefined.

(B) -0.648

(C) -0.078

(D) 0

6)



Graph of f

The figure above shows the graph of a function f, which has a vertical tangent at $x = 2$ and a horizontal tangent at $x = 4$. Which of the following statements is false?

(A) f is not differentiable at $x = -1$ because the graph of f has a jump discontinuity at $x = -1$.

(B) f is not differentiable at $x = 2$ because the graph of f has a vertical tangent at $x = 2$.

(C) f is not differentiable at $x = 4$ because the graph of f has a horizontal tangent at $x = 4$.

(D) f is not differentiable at $x = 6$ because the graph of f has a removable discontinuity at $x = 6$.

7)

$$f(x) = \begin{cases} x^2 - 20 & \text{for } x < 5 \\ -x^2 + 20 & \text{for } x \geq 5 \end{cases}$$

Let f be the function defined above. Which of the following statements is true?

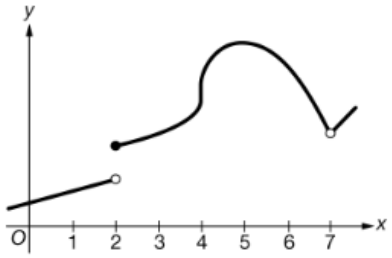
(A) f is not differentiable at $x = 5$ because f is not continuous at $x = 5$.

(B) f is not differentiable at $x = 5$ because the graph of f has a sharp corner at $x = 5$.

(C) f is not differentiable at $x = 5$ because the graph of f has a vertical tangent at $x = 5$.

(D) f is not differentiable at $x = 5$ because f is not defined at $x = 5$.

8)



Graph of f

The figure above shows the graph of a function f, which has a vertical tangent at $x = 4$ and a horizontal tangent at $x = 5$. Which of the following statements is false?

(A) f is not differentiable at $x = 2$ because the graph of f has a jump discontinuity at $x = 2$.

(B) f is not differentiable at $x = 4$ because the graph of f has a vertical tangent at $x = 4$.

(C) f is not differentiable at $x = 5$ because the graph of f has a horizontal tangent at $x = 5$.

(D) f is not differentiable at $x = 7$ because the graph of f has a removable discontinuity at $x = 7$.

9) $g(x) = \begin{cases} 2 - 2x & \text{for } x < 1 \\ 5x - 5 & \text{for } x \geq 1 \end{cases}$

If g is the function defined above, then $g'(1)$ is

(A) -2

(B) 0

(C) 5

(D) nonexistent

10) What is the value of $\lim_{h \to 0} \frac{(16+h)^{\frac{1}{4}} - 2}{h}$?

(A) 0

(B) $\frac{1}{32}$

(C) $\frac{1}{8}$

(D) 1

11) $\lim_{h\to 0}\frac{(x+h)^2+4(x+h)-x^2-4x}{h}$ is

(A) x^3+x^4

(B) x^2+4x

(C) $2x+4$

(D) nonexistent

12) Let f and g be differentiable functions such that $f'(0)=3$ and $g'(0)=7$. If $h(x)=3f(x)-2g(x)-5\cos x-3$, what is the value of $h'(0)$?

(A) -8

(B) -5

(C) 1

(D) 28

13) Let f be the function given by $f(x)=5x^3-3x-7$. What is the value of $f'(-2)$?

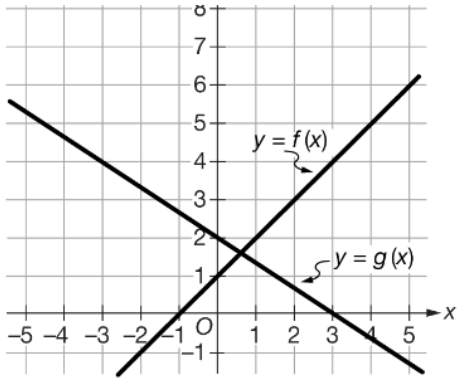
(A) -114

(B) 17

(C) 50

(D) 57

14)



The graphs of the linear functions f and g are shown above. If $h(x) = f(x) + g(x)$, then $h'(x) =$

(A) $\frac{5}{3}$

(B) 0

(C) $\frac{1}{3}$

(D) $\frac{1}{3}x + 3$

15) If $f(x) = \sin x$, then $\lim_{x \to 2\pi} \frac{f(2\pi) - f(x)}{x - 2\pi} =$

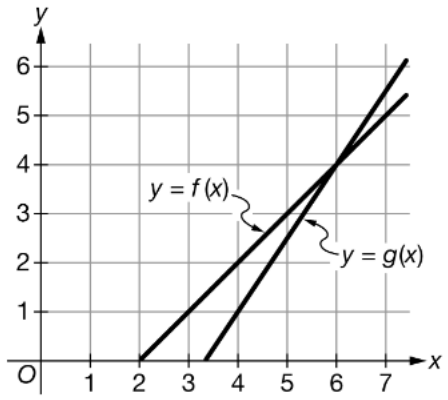
(A) -2π

(B) -1

(C) 1

(D) 2π

16)



The graphs of the linear function f and the linear function g are shown in the figure above. If $h(x) = f(x)g(x)$, then $h'(4) =$

(A) -2

(B) $\frac{3}{2}$

(C) $\frac{7}{2}$

(D) 4

17) Let f be a differentiable function such that $f(2) = 2$ and $f'(2) = 5$. If $g(x) = x^3 f(x)$, what is the value of $g'(2)$?

(A) 17

(B) 24

(C) 60

(D) 64

18)

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
0	4	$\frac{1}{2}$	-2	$\frac{3}{2}$

The table above gives values of the differentiable functions f and g and their derivatives at $x = 0$. If $h(x) = \frac{6f(x)}{g(x)-1}$, then $h'(0) =$

(A) 15

(B) 3

(C) 2

(D) -5

19) Let f be a differentiable function such that $f(8) = 2$ and $f'(8) = 5$. If g is the function defined by $g(x) = \frac{f(x)}{\sqrt[3]{x}}$, what is the value of $g'(8)$?

(A) $\frac{59}{24}$

(B) $\frac{5}{2}$

(C) $\frac{61}{24}$

(D) 60

20) If $f(x) = \sec x$, then $\lim_{x \to \frac{\pi}{3}} \frac{f(x) - f\left(\frac{\pi}{3}\right)}{x - \frac{\pi}{3}}$ is

(A) 0

(B) $\sec\left(\frac{\pi}{3}\right)$

(C) $\sec\left(\frac{\pi}{3}\right)\tan\left(\frac{\pi}{3}\right)$

(D) nonexistent

21) $\frac{d}{dx}(\cos x \tan x) =$

(A) $\sec x + \sin x \tan x$

(B) $\cos x$

(C) $-\sin x \sec^2 x$

(D) $\sin x$

22) Which of the following correctly shows the derivation of $\frac{d}{dx}(\cot x)$?

(A) $\frac{d}{dx}(\cot x) = \frac{d}{dx}\left(\frac{1}{\tan x}\right) = \frac{1}{\frac{d}{dx}(\tan x)} = \frac{-1}{\sec^2 x}$

(B) $\frac{d}{dx}(\cot x) = \frac{d}{dx}\left(\frac{1}{\tan x}\right) = \frac{1}{\frac{d}{dx}(\tan x)} = \frac{1}{\sec^2 x}$

(C) $\frac{d}{dx}(\cot x) = \frac{d}{dx}\left(\frac{1}{\tan x}\right) = \frac{\tan x \frac{d}{dx}(1) - 1 \cdot \frac{d}{dx}(\tan x)}{\tan^2 x} = \frac{(\tan x) \cdot 0 - \sec^2 x}{\tan^2 x} = -\frac{\sec^2 x}{\tan^2 x}$

(D) $\frac{d}{dx}(\cot x) = \frac{d}{dx}\left(\frac{1}{\tan x}\right) = \frac{\tan x \frac{d}{dx}(1) + 1 \cdot \frac{d}{dx}(\tan x)}{\tan^2 x} = \frac{\tan x \cdot 0 + \sec^2 x}{\tan^2 x} = \frac{\sec^2 x}{\tan^2 x}$

23) Let f be the function defined by $f(x) = \sin(h(x))$, where h is a differentiable function. Which of the following is equivalent to the derivative of f with respect to x ?

(A) $\cos(h(x))$

(B) $\cos(h\prime(x))$

(C) $\cos(h(x))h\prime(x)$

(D) $\sin(h(x))h\prime(x)$

24) Let $f(x) = x^3$ and $g(x) = \frac{x}{x-1}$. If h is the function defined by $h(x) = f(g(x))$, which of the following gives a correct expression for $h'(x)$?

(A) $3(g(x))^2 = 3\left(\frac{x}{x-1}\right)^2$

(B) $3(g'(x))^2 = 3\left(\frac{(x-1)-x}{(x-1)^2}\right)^2$

(C) $3(g(x))^2 g'(x) = 3\left(\frac{x}{x-1}\right)^2 \cdot \frac{(x-1)-x}{(x-1)^2}$

(D) $(g'(x))^3 = \left(\frac{(x-1)-x}{(x-1)^2}\right)^3$

25) Let g be the function given by $g(x) = \sin(-x) + \cos x - 10$. Which of the following statements is true for $y = g(x)$?

(A) $y + 10 = \frac{d^4y}{dx^4}$

(B) $g^{(4)}(x) = (g''(x))^2$

(C) $g''(x) - 10 = g(x)$

(D) $y'' = \sin(-x) + \cos x$