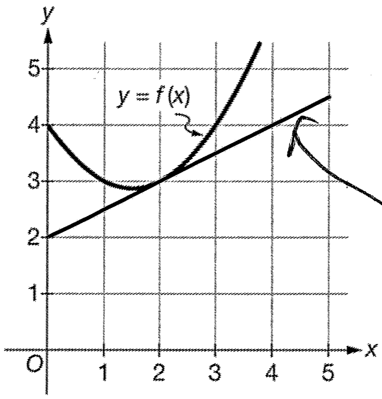
Unit 2 AP Classroom Practice for Sections 1-5

1)



slope of this line

Shown above is the graph of the differentiable function f, along with the line tangent to the graph of f at $x = 2$. What is the value of $f'(2)$?

(A) $\frac{1}{2}$

(B) 2

(C) 3

(D) 4

2) An equation for the line tangent to the graph of the differentiable function f at $x = 3$ is $y = 4x + 6$. Which of the following statements must be true?

~~I.~~ $f(0) = 6$ ← on tangent line, not the function

II. $f(3) = 18$ ← on function and tangent line

III. $f'(3) = 4$ ← slope of tangent line

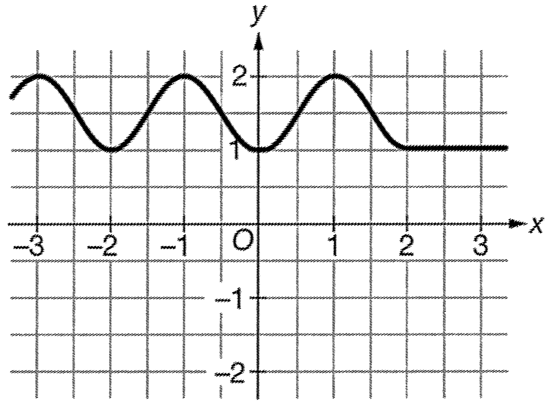
(A) None

(B) I and II only

(C) II and III only

(D) I, II, and III

3)



Graph of f'

slope of tangent line at $x=1$

$f'(1) = 2$

from graph above

Let f be a differentiable function with $f(1) = 3$. The graph of f', the derivative of f, is shown above.

Which of the following statements is true about the line tangent to the graph of f at $x = 1$?

(A) The tangent line has slope 2 and passes through the point $(1, 3)$.

(B) The tangent line has slope 2 and passes through the point $(1, 2)$.

(C) The tangent line has slope 0 and passes through the point $(1, 3)$.

(D) The tangent line has slope 0 and passes through the point $(1, 2)$.

4) GRAPHING CALCULATOR NEEDED (see where $f(x)$ is increasing and steepest!)

Let f be the function given by $f(x) = x^4 + \frac{1}{2}x^3 - 5x^2 + \tan\left(\frac{x}{2}\right)$. Of the following values of x, at which does the line tangent to the graph of f have the greatest slope?

(A) $x = -2$

(B) $x = -1$

(C) $x = 0$

(D) $x = 1$

or

$f'(x) = 4x^3 + \frac{3}{2}x^2 - 10x + \sec^2\left(\frac{x}{2}\right) \cdot \frac{1}{2}$

$f'(-1) \approx 8.149$ which is the LARGEST

5) GRAPHING CALCULATOR NEEDED (Find with calculator)

Let f be the function given by $f(x) = \cos x - \csc x$. What is the value of $f'(1)$?

(A) $f'(1)$ is undefined.

(B) -0.648

(C) -0.078

(D) 0

or

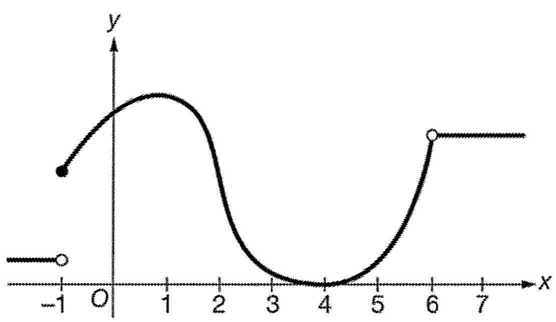
$f'(x) = -\sin x + \csc x \cot x$

$f'(1) = -\sin(1) + \csc(1)\cot(1)$

use calculator

$= -.078$

6)



Graph of f

The figure above shows the graph of a function f, which has a vertical tangent at $x = 2$ and a horizontal tangent at $x = 4$. Which of the following statements is false?

(A) f is not differentiable at $x = -1$ because the graph of f has a jump discontinuity at $x = -1$.

(B) f is not differentiable at $x = 2$ because the graph of f has a vertical tangent at $x = 2$.

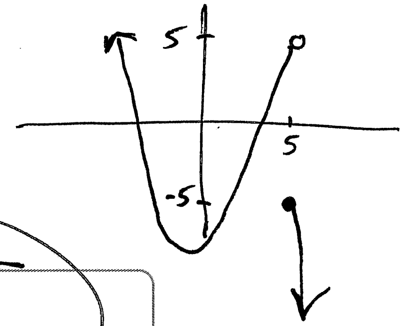
(C) f is not differentiable at $x = 4$ because the graph of f has a horizontal tangent at $x = 4$

has a slope of zero so "IS" differentiable

(D) f is not differentiable at $x = 6$ because the graph of f has a removable discontinuity at $x = 6$.

7)

$$f(x) = \begin{cases} x^2 - 20 & \text{for } x < 5 \\ -x^2 + 20 & \text{for } x \geq 5 \end{cases}$$



Let f be the function defined above. Which of the following statements is true?

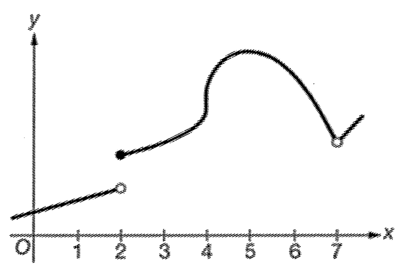
(A) f is not differentiable at $x = 5$ because f is not continuous at $x = 5$.

(B) f is not differentiable at $x = 5$ because the graph of f has a sharp corner at $x = 5$.

(C) f is not differentiable at $x = 5$ because the graph of f has a vertical tangent at $x = 5$.

(D) f is not differentiable at $x = 5$ because f is not defined at $x = 5$.

8)



Graph of f

The figure above shows the graph of a function f, which has a vertical tangent at $x = 4$ and a horizontal tangent at $x = 5$. Which of the following statements is false?

(A) f is not differentiable at $x = 2$ because the graph of f has a jump discontinuity at $x = 2$
.

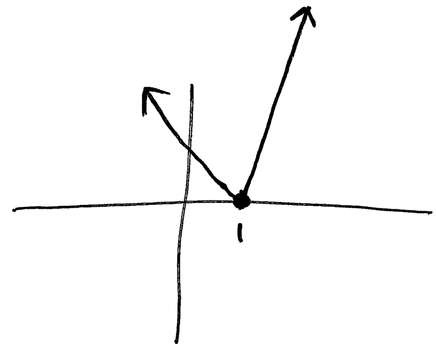
(B) f is not differentiable at $x = 4$ because the graph of f has a vertical tangent at $x = 4$.

(C) f is not differentiable at $x = 5$ because the graph of f has a horizontal tangent at $x = 5$

slope of zero so is differentiable

(D) f is not differentiable at $x = 7$ because the graph of f has a removable discontinuity at $x = 7$.

9) $g(x) = \begin{cases} 2-2x & \text{for } x<1 \\ 5x-5 & \text{for } x \geq 1 \end{cases}$



If g is the function defined above, then $g'(1)$ is

Ⓐ -2

Ⓑ 0

Ⓒ 5

Ⓓ nonexistent

slope from left NOT equal slope from right

10) What is the value of $\lim\limits_{h \to 0} \frac{(16+h)^{\frac{1}{4}}-2}{h}$?

Ⓐ 0

Ⓑ $\frac{1}{32}$

Ⓒ $\frac{1}{8}$

Ⓓ 1

means derivative of $\sqrt[4]{x}$ evaluated when $x=16$

If $f(x) = x^{1/4}$

$f'(x) = \frac{1}{4}x^{-3/4}$

$f'(16) = \frac{1}{4(\sqrt[4]{16})^3}$

$= \frac{1}{4 \cdot 2^3}$

$= \frac{1}{4 \cdot 8}$

$= \frac{1}{32}$

11) $\lim_{h \to 0} \frac{(x+h)^2+4(x+h)-x^2-4x}{h}$ is

means the derivative of "x^2+4x"

if $f(x) = x^2+4x$

$f'(x) = 2x+4$

(A) $x^3 + x^4$

(B) $x^2 + 4x$

(C) $2x + 4$

(D) nonexistent

12) Let f and g be differentiable functions such that $f'(0) = 3$ and $g'(0) = 7$. If $h(x) = 3f(x) - 2g(x) - 5\cos x - 3$, what is the value of $h'(0)$?

$h'(x) = 3 \cdot f'(x) - 2 \cdot g'(x) + 5\sin x$

$h'(0) = 3 \cdot f'(0) - 2 \cdot g'(0) + 5\sin(0)$

$= 3 \cdot 3 - 2 \cdot 7 + 5 \cdot 0$

$= 9 - 14$

$= -5$

(A) -8

(B) -5

(C) 1

(D) 28

13) Let f be the function given by $f(x) = 5x^3 - 3x - 7$. What is the value of $f'(-2)$?

$f'(x) = 15x^2 - 3 \Rightarrow f'(-2) = 15 \cdot (-2)^2 - 3$

$= 15 \cdot 4 - 3$

$= 57$

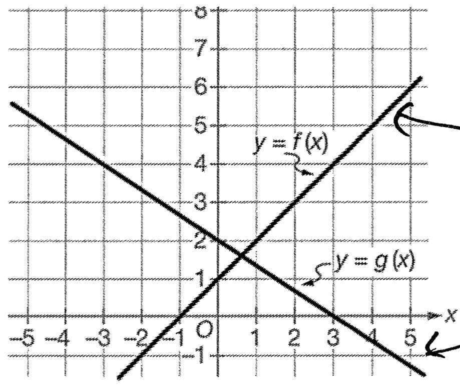
(A) -114

(B) 17

(C) 50

(D) 57

14)



f'(x) is always "1"

&

g'(x) is always "$-\frac{2}{3}$"

The graphs of the linear functions f and g are shown above. If $h(x) = f(x) + g(x)$, then $h'(x) =$ $f'(x) + g'(x)$

$1 + -\frac{2}{3}$

$= \frac{1}{3}$

(A) $\frac{5}{3}$

(B) 0

(C) $\frac{1}{3}$

(D) $\frac{1}{3}x + 3$

Tricky

15) If $f(x) = \sin x$, then $\lim_{x \to 2\pi} \frac{f(2\pi) - f(x)}{x - 2\pi} =$

alternate def'n of limit def'n of derivative
BUT NUMERATOR OUT OF ORDER (f(x) should be 1st)

means opposite of the derivative of sinx evaluated at $x = 2\pi$

(A) -2π

(B) -1

(C) 1

(D) 2π

if $f(x) = \sin x$

$f'(x) = \cos x$

$f'(2\pi) = \cos(2\pi)$

$= 1$

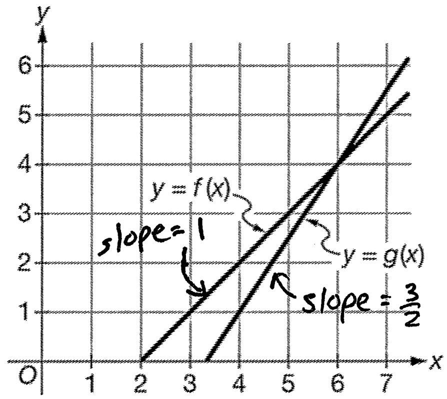
so... "-1"

opposite of derivative of sinx when $x = 2\pi$

$= \lim_{x \to 2\pi} \frac{-[f(x) - f(2\pi)]}{x - 2\pi}$

$= -\lim_{x \to 2\pi} \frac{f(x) - f(2\pi)}{x - 2\pi}$

16)



$h'(x) = f(x)\cdot g'(x) + g(x)\cdot f'(x)$

$h'(4) = f(4)\cdot g'(4) + g(4)\cdot f'(4)$

$= 2 \cdot \frac{3}{2} + 1 \cdot 1$

$= 3 + 1$

$= 4$

The graphs of the linear function f and the linear function g are shown in the figure above. If $h(x) = f(x)g(x)$, then $h'(4) =$

(A) -2

(B) $\frac{3}{2}$

(C) $\frac{7}{2}$

(D) 4

17) Let f be a differentiable function such that $f(2) = 2$ and $f'(2) = 5$. If $g(x) = x^3 f(x)$, what is the value of $g'(2)$?

$g'(x) = x^3 \cdot f'(x) + f(x) \cdot 3x^2$

$g'(2) = 2^3 \cdot f'(2) + f(2) \cdot 3(2)^2$

$8 \cdot 5 + 2 \cdot 12$

$40 + 24 = 64$

(A) 17

(B) 24

(C) 60

(D) 64

18)

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
0	4	$\frac{1}{2}$	-2	$\frac{3}{2}$

The table above gives values of the differentiable functions f and g and their derivatives at $x = 0$. If $h(x) = \frac{6f(x)}{g(x)-1}$, then $h'(0) =$

(A) 15

(B) 3

(C) 2

(D) -5

$$h'(x) = \frac{(g(x)-1)\cdot 6\cdot f'(x) - 6f(x)\cdot g'(x)}{[g(x)-1]^2}$$

$$h'(0) = \frac{(g(0)-1)\cdot 6\cdot f'(0) - 6f(0)\cdot g'(0)}{[g(0)-1]^2}$$

$$= \frac{(-2-1)\cdot 6\cdot \frac{1}{2} - 6\cdot 4\cdot \frac{3}{2}}{[-2-1]^2}$$

$$= \frac{-3\cdot 3 - 36}{(-3)^2}$$

$$= \frac{-9-36}{9}$$

$$= \frac{-45}{9}$$

$$= -5$$

19) Let f be a differentiable function such that $f(8) = 2$ and $f'(8) = 5$. If g is the function defined by $g(x) = \frac{f(x)}{\sqrt[3]{x}}$, what is the value of $g'(8)$?

(A) $\frac{59}{24}$

(B) $\frac{5}{2}$

(C) $\frac{61}{24}$

(D) 60

$$g'(x) = \frac{\sqrt[3]{x}\cdot f'(x) - f(x)\cdot \frac{1}{3}x^{-2/3}}{(\sqrt[3]{x})^2}$$

$$g'(8) = \frac{\sqrt[3]{8}\cdot f'(8) - f(8)\cdot \frac{1}{3}\cdot(8)^{-2/3}}{(\sqrt[3]{8})^2}$$

$$= \frac{2\cdot 5 - 2\cdot \frac{1}{3}\cdot\frac{1}{4}}{2^2} \Rightarrow \frac{10-\frac{1}{6}}{4} = \frac{\frac{59}{6}}{4} = \frac{59}{24}$$

20) If $f(x) = \sec x$, then $\lim_{x \to \frac{\pi}{3}} \frac{f(x) - f(\frac{\pi}{3})}{x - \frac{\pi}{3}}$ is

means derivative of "$\sec x$" evaluated at $x = \frac{\pi}{3}$

Ⓐ 0

Ⓑ $\sec\left(\frac{\pi}{3}\right)$

Ⓒ $\sec\left(\frac{\pi}{3}\right)\tan\left(\frac{\pi}{3}\right)$

Ⓓ nonexistent

If $f(x) = \sec x$

$f'(x) = \sec x \tan x$

$f'\left(\frac{\pi}{3}\right) = \sec\frac{\pi}{3}\tan\frac{\pi}{3}$

$= \frac{1}{\frac{1}{2}} \cdot \sqrt{3}$

$= 2 \cdot \sqrt{3}$

21) $\frac{d}{dx}(\cos x \tan x) =$

Ⓐ $\sec x + \sin x \tan x$

Ⓑ $\cos x$

Ⓒ $-\sin x \sec^2 x$

Ⓓ $\sin x$

$\frac{d}{dx} \cos x \tan x = \cos x \sec^2 x + \tan x \cdot (-\sin x)$

$= \sec x - \tan x \sin x$

$= \frac{1}{\cos x} - \frac{\sin x}{\cos x} \cdot \sin x$

$= \frac{1 - \sin^2 x}{\cos x}$

$= \frac{\cos^2 x}{\cos x}$

$= \cos x$

22) Which of the following correctly shows the derivation of $\frac{d}{dx}(\cot x)$?

Ⓐ $\frac{d}{dx}(\cot x) = \frac{d}{dx}\left(\frac{1}{\tan x}\right) = \frac{1}{\frac{d}{dx}(\tan x)} = \frac{-1}{\sec^2 x}$

Ⓑ $\frac{d}{dx}(\cot x) = \frac{d}{dx}\left(\frac{1}{\tan x}\right) = \frac{1}{\frac{d}{dx}(\tan x)} = \frac{1}{\sec^2 x}$

QUOTIENT RULE

Ⓒ $\frac{d}{dx}(\cot x) = \frac{d}{dx}\left(\frac{1}{\tan x}\right) = \frac{\tan x \frac{d}{dx}(1) - 1 \cdot \frac{d}{dx}(\tan x)}{\tan^2 x} = \frac{(\tan x) \cdot 0 - \sec^2 x}{\tan^2 x} = -\frac{\sec^2 x}{\tan^2 x}$

Ⓓ $\frac{d}{dx}(\cot x) = \frac{d}{dx}\left(\frac{1}{\tan x}\right) = \frac{\tan x \frac{d}{dx}(1) + 1 \cdot \frac{d}{dx}(\tan x)}{\tan^2 x} = \frac{\tan x \cdot 0 + \sec^2 x}{\tan^2 x} = \frac{\sec^2 x}{\tan^2 x}$

23) Let f be the function defined by $f(x) = \sin(h(x))$, where h is a differentiable function. Which of the following is equivalent to the derivative of f with respect to x ? $f'(x) = \cos(h(x)) \cdot h'(x)$

(A) $\cos(h(x))$

(B) $\cos(h\prime(x))$

(C) $\cos(h(x))h\prime(x)$ CHAIN RULE

(D) $\sin(h(x))h\prime(x)$

24) Let $f(x) = x^3$ and $g(x) = \frac{x}{x-1}$. If h is the function defined by $h(x) = f(g(x))$, which of the following gives a correct expression for $h'(x)$?

(A) $3(g(x))^2 = 3\left(\frac{x}{x-1}\right)^2$

(B) $3(g'(x))^2 = 3\left(\frac{(x-1)-x}{(x-1)^2}\right)^2$

(C) $3(g(x))^2 g'(x) = 3\left(\frac{x}{x-1}\right)^2 \cdot \frac{(x-1)-x}{(x-1)^2}$

(D) $(g'(x))^3 = \left(\frac{(x-1)-x}{(x-1)^2}\right)^3$

$h(x) = \left(\frac{x}{x-1}\right)^3$

$h'(x) = 3\left(\frac{x}{x-1}\right)^2 \cdot \frac{(x-1)\cdot 1 - x \cdot 1}{(x-1)^2}$

$= 3\left(\frac{x}{x-1}\right)^2 \cdot \frac{(x-1)-x}{(x-1)^2}$

25) Let g be the function given by $g(x) = \sin(-x) + \cos x - 10$. Which of the following statements is true for $y = g(x)$?

or y

(A) $y + 10 = \frac{d^4y}{dx^4}$ so... $\frac{d^4y}{dx^4} = \underbrace{\sin(-x) + \cos x - 10}_{\text{"}y\text{"}} + 10 = \sin(-x) + \cos x$

(B) $g^{(4)}(x) = (g''(x))^2$ $g^4(x) = (-\sin(-x) - \cos x)^2$ NOT equal to $g^4(x)$ below!

(C) $g''(x) - 10 = g(x)$ $-\sin(-x) - \cos x - 10 \neq g(x)$

(D) $y'' = \sin(-x) + \cos x$ NOT TRUE (see $g''(x)$ below)

$g'(x) = -\cos(-x) - \sin x$

$g''(x) = -\sin(-x) - \cos x$

$g^3(x) = \cos(-x) + \sin x$

$g^4(x) = \sin(-x) + \cos x$