Unit 2 AP Classroom Practice for Sections 6-9

1) $\lim_{x\to 0} \frac{5^x-1}{x} =$ means derivative of $y=5^x$ evaluated when $x=0$ (so ... slope of 5^x graph at $x=0$)

$y' = 5^x \cdot \ln 5$

$y'(0) = 5^0 \cdot \ln 5$

$= 1 \cdot \ln 5$

$= \ln 5$

Ⓐ 0

Ⓑ $\ln 5$

Ⓒ $5 \ln 5$

Ⓓ 1

out of order for alternate version of limit def'n of derivative

2) If $f(x) = \ln x$, then $\lim_{x\to 2} \frac{f(2)-f(x)}{x-2} =$ $- \lim_{x\to 2} \frac{f(x)-f(2)}{x-2} = - \lim_{x\to 2} \frac{\ln x - \ln 2}{x-2}$

means the opposite of the derivative of $\ln x$ evaluated at $x=2$

Ⓐ $-\ln 2$

Ⓑ $-\frac{1}{2}$

Ⓒ $\frac{1}{2}$

Ⓓ $\ln 2$

$y = \ln x$

$y' = \frac{1}{x}$ $\quad y'(2) = \frac{1}{2}$ $\quad$ so answer is $-\frac{1}{2}$

3) Let f be the function given by $f(x) = 2\cos x + 3e^x$. What is the value of $f'(0)$?

(A) -3

(B) 0

(C) 3

(D) 5

$f'(x) = -2\sin x + 3e^x$

$f'(0) = -2 \cdot \sin(0) + 3e^0$

$= -2 \cdot 0 + 3 \cdot 1$

$= 0 + 3$

Product Rule

4) If $f(x) = e^x \sin x$, then $f'(x) =$

(A) $e^x \cos x$

(B) $-e^x \cos x$

(C) $e^x(\sin x + \cos x)$

(D) $e^x(\sin x - \cos x)$

$f'(x) = e^x \cdot \cos x + \sin x \cdot e^x$

or

$e^x(\cos x + \sin x)$

Product Rule!

5) If $f(x) = \ln x \cos x$, then $f'(x) =$ $\ln x \cdot (-\sin x) + \cos x \cdot \frac{1}{x}$

(A) $-\frac{\sin x}{x}$

(B) $\frac{\sin x}{x}$

(C) $\frac{\cos x}{x} - \ln x \sin x$

(D) $\frac{\cos x}{x} + \ln x \sin x$

Quotient Rule

6) If $f(x) = \frac{\cos x}{\ln x}$, then $f'(x) =$ $\frac{\ln x \cdot (-\sin x) - \cos x \cdot \frac{1}{x}}{(\ln x)^2}$

(A) $\frac{x \sin x \ln x - \cos x}{x(\ln x)^2}$

(B) $\frac{-x \sin x \ln x - \cos x}{x(\ln x)^2}$

(C) $-x \sin x$

(D) $\frac{-x \sin x \ln x + \cos x}{x(\ln x)^2}$

$$\frac{-\ln x \cdot (\sin x) - \frac{\cos x}{x}}{(\ln x)^2} \cdot \frac{x}{x}$$

$$\frac{-x \ln x \cdot (\sin x) - \cos x}{x \cdot (\ln x)^2}$$

Quotient Rule

7) If $f(x) = \frac{\sin x}{e^x}$, then $f'(x) =$ $\frac{e^x \cdot \cos x - \sin x \cdot e^x}{(e^x)^2} = \frac{\cancel{e^x}(\cos x - \sin x)}{(e^x)^{\cancel{2}\,1}}$

(A) $\frac{-\cos x - \sin x}{e^x}$

(B) $\frac{\cos x - \sin x}{e^x}$

(C) $\frac{\sin x - \cos x}{e^x}$

(D) $\frac{\cos x + \sin x}{e^x}$

$f'(x) = 3\left[\cos\sqrt{x} - \ln(x^2)\right]^2 \cdot \left[-\sin\sqrt{x} \cdot \frac{1}{2}x^{-1/2} - \frac{1}{x^2} \cdot 2x\right]$

$= 3\left(\cos\sqrt{x} - \ln(x^2)\right)^2 \cdot \left[\frac{-\sin\sqrt{x}}{2\sqrt{x}} - \frac{2}{x}\right]$

Chain Rule

8) If $f(x) = \left(\cos\left(\sqrt{x}\right) - \ln\left(x^2\right)\right)^3$, then $f'(x) =$

$\hookrightarrow f(x) = \left(\cos(x^{1/2}) - \ln(x^2)\right)^3$

(A) $3\left(-\frac{1}{2\sqrt{x}}\sin\left(\sqrt{x}\right) - \frac{2}{x}\right)^2$

(B) $3\left(\cos\left(\sqrt{x}\right) - \ln\left(x^2\right)\right)^2 \cdot \left(-\sin\left(\sqrt{x}\right) - \frac{1}{x^2}\right)$

(C) $3\left(\cos\left(\sqrt{x}\right) - \ln\left(x^2\right)\right)^2 \cdot \left(\frac{1}{2\sqrt{x}}\cos\left(\sqrt{x}\right) - \frac{2}{x}\right)$

(D) $3\left(\cos\left(\sqrt{x}\right) - \ln\left(x^2\right)\right)^2 \cdot \left(-\frac{1}{2\sqrt{x}}\sin\left(\sqrt{x}\right) - \frac{2}{x}\right)$

Chain Rule 9) If $f(x) = \left(e^{3x} + \sin(2x)\right)^4$, then $f'(x) =$ $4\left(e^{3x} + \sin(2x)\right)^3 \cdot \left[e^{3x} \cdot 3 + \cos(2x) \cdot 2\right]$

(A) $4\left(3e^{3x} + 2\cos(2x)\right)^3$

(B) $4\left(e^{3x} + \sin(2x)\right)^3\left(e^{3x} + \cos(2x)\right)$

(C) $4\left(e^{3x} + \sin(2x)\right)^3\left(3e^{3x} + 2\sin(2x)\right)$

(D) $4\left(e^{3x} + \sin(2x)\right)^3\left(3e^{3x} + 2\cos(2x)\right)$

10) Let f be the function defined by $f(x) = e^{h(x)}$, where h is a differentiable function. Which of the following is equivalent to the derivative of f with respect to x ? $f'(x) = e^{h(x)} \cdot h'(x)$

Chain Rule

(A) $e^{h(x)}$

(B) $e^{h'(x)}$

(C) $h'(x)e^{h(x)}$

(D) $h(x)e^{h(x)-1}$

$$e^{2y}\cdot 2\frac{dy}{dx} - e^{(y^2-y)}\cdot(2y-1)\frac{dy}{dx} = 4x^3-2x$$

OUCH!

$$\frac{dy}{dx}\left(2e^{2y} - (2y-1)\cdot e^{y^2-y}\right) = 4x^3-2x$$

11) If $e^{2y} - e^{(y^2-y)} = x^4 - x^2$, then the value of $\frac{dy}{dx}$ at the point $(1, 0)$ is $\frac{dy}{dx} = \frac{4x^3-2x}{2e^{2y}-(2y-1)\cdot e^{y^2-y}}$

$\frac{dy}{dx}(1,0)$

$= \frac{4-2}{2-(-1)\cdot 1}$

$= \frac{2}{2+1}$

$= \frac{2}{3}$

Ⓐ 0

Ⓑ $\frac{1}{2}$

Ⓒ $\frac{2}{3}$

Ⓓ 2

OUCH!!

12) If $y = \ln(2x^2 - 3y^2)$, then $\frac{dy}{dx} =$ $\frac{dy}{dx} = \frac{1}{2x^2-3y^2}\cdot\left(4x - 6y\frac{dy}{dx}\right)$

Ⓐ $\frac{1}{2x^2-3y^2}$

Ⓑ $\frac{4x}{2x^2-3y^2}$

Ⓒ $\frac{4x-6y}{2x^2-3y^2}$

Ⓓ $\frac{4x}{2x^2-3y^2+6y}$

$$\frac{dy}{dx} = \frac{4x}{2x^2-3y^2} - \frac{6y}{2x^2-3y^2}\frac{dy}{dx}$$

$$\frac{6y}{2x^2-3y^2}\frac{dy}{dx} + 1\frac{dy}{dx} = \frac{4x}{2x^2-3y^2}$$

$$\frac{dy}{dx}\left(\frac{6y}{2x^2-3y^2}+1\right) = \frac{4x}{2x^2-3y^2} \rightarrow \frac{dy}{dx} = \frac{\frac{4x}{2x^2-3y^2}}{\frac{6y}{2x^2-3y^2}+1}$$

$$\frac{dy}{dx} = \frac{\frac{4x}{2x^2-3y^2}}{\frac{6y}{2x^2-3y^2}+1}\cdot\frac{(2x^2-3y^2)}{(2x^2-3y^2)} \quad \boxed{\frac{dy}{dx} = \frac{4x}{6y+2x^2-3y^2}}$$

$$2x \cdot 2y\frac{dy}{dx} + 2y^2 - \left[3x^2\frac{dy}{dx} + 6xy\right] = 6$$

$$4xy\frac{dy}{dx} + 2y^2 - 3x^2\frac{dy}{dx} - 6xy = 6$$

13) If $2xy^2 - 3x^2y = 6x$, then $\frac{dy}{dx} =$

(A) $\frac{6}{4y-6x}$

(B) $\frac{2y^2-2xy-6}{3x^2}$

(C) $\frac{-2y^2+6xy+6}{4xy-3x^2}$

(D) $\frac{-2y^2+6xy+3x^2+6}{4xy}$

$$4xy\frac{dy}{dx} - 3x^2\frac{dy}{dx} = 6 + 6xy - 2y^2$$

$$\frac{dy}{dx}(4xy - 3x^2) = 6 + 6xy - 2y^2$$

$$\frac{dy}{dx} = \frac{6 + 6xy - 2y^2}{4xy - 3x^2}$$

14) If $f(x) = \arcsin x$, then $\lim\limits_{x \to \frac{1}{2}} \frac{f(x) - f(\frac{1}{2})}{x - \frac{1}{2}}$ is

← means derivative (slope) of arcsinx when $x = 1/2$

(A) 0

(B) $\frac{\pi}{6}$

(C) $\frac{2}{\sqrt{3}}$

(D) nonexistent

$$f'(x) = \frac{1}{\sqrt{1-x^2}}$$

$$f'(\tfrac{1}{2}) = \frac{1}{\sqrt{1-(1/2)^2}} \qquad f'(\tfrac{1}{2}) = \frac{1}{\sqrt{1-\frac{1}{4}}} \qquad f'(\tfrac{1}{2}) = \frac{1}{\sqrt{\frac{3}{4}}} = \frac{1}{\left(\frac{\sqrt{3}}{2}\right)}$$

$$= \frac{2}{\sqrt{3}}$$

MORE COMMON ANSWER

15) $\frac{d}{dx}\left(\csc^{-1}(e^x)\right) =$ $\dfrac{-1}{|e^x|\sqrt{(e^x)^2-1}} \cdot e^x = \dfrac{-\cancel{e^x}}{\cancel{e^x}\sqrt{e^{2x}-1}} = \boxed{\dfrac{-1}{\sqrt{e^{2x}-1}}}$

(A) $-e^x \cdot \cot(e^x) \cdot \csc(e^x)$

(B) $e^x \cdot \cot(e^x) \cdot \csc^3(e^x)$

(C) $\frac{e^x}{\sqrt{1-e^{2x}}}$

(D) $\frac{-1}{e^x\sqrt{1-\left(\frac{1}{e^{2x}}\right)}}$

NOTE: $\sqrt{1-\frac{1}{e^{2x}}} = \sqrt{\frac{e^{2x}-1}{e^{2x}}} = \frac{\sqrt{e^{2x}-1}}{\sqrt{e^{2x}}} = \frac{\sqrt{e^{2x}-1}}{e^x}$

$\frac{d}{dx}(\csc^{-1} e^x) = \frac{d}{dx}\left(\sin^{-1}\left(\frac{1}{e^x}\right)\right)$

They used this one!

So... $\dfrac{-1}{\dfrac{\cancel{e^x}\sqrt{e^{2x}-1}}{\cancel{e^x}}}$

16) $\frac{d}{dx}\left(\tan^{-1}(3x)\right) =$

(A) $3\sec^2(3x)$

(B) $-3\csc^2(3x)$

(C) $\frac{3}{\sqrt{1-(3x)^2}}$

(D) $\frac{3}{1+(3x)^2}$

$\frac{d}{dx}\left(\mathrm{Tan}^{-1}(3x)\right) = \frac{1}{1+(3x)^2} \cdot 3$

$$\frac{1}{1+(x-2y+2)^2} \cdot \left(1-2\frac{dy}{dx}\right) = 2x - 3\frac{dy}{dx}$$

NOTE:
$\tan^{-1}(2)$ is just a number so its derivative is zero!

17) Which of the following could be used to find the slope of the line tangent to the curve $\tan^{-1}(x-2y+2) = x^2 - 3y + \tan^{-1}(2) - 1$?

(A) $\frac{1}{1+(x-2y+2)^2} = 2x - 3\frac{dy}{dx}$

(B) $\frac{-1}{1+(x-2y+2)^2} = 2x - 3$

(C) $\frac{1-2\frac{dy}{dx}}{1+(x-2y+2)^2} = 2x - 3\frac{dy}{dx}$

(D) $\frac{1-2\frac{dy}{dx}}{1+(x-2y+2)} = 2x - 3\frac{dy}{dx}$

18) Which of the following does not require the use of the chain rule to find $\frac{dy}{dx}$?

(A) $y = \sin^{-1}(3x^2 - 4)$

NOTE: when diff "y" with respect to x we say times $\frac{dy}{dx}$ and is actually because of the chain rule

(B) $3x^6 - 4y^2 = 2xy^5 + 7$

(C) $y = 3x^2 - \sqrt{x} + \frac{2}{x}$ $\quad y = 3x^2 - x^{1/2} + 2x^{-1}$

(D) $\cos(x+y) + 2^y - x = 0$

Only power rule needed!

19) Let f be the function given by $f(x) = \sin x + e^{-x} + 3x$. Which of the following statements is true for $y = f(x)$?

(A) $y'' = \sin x + e^{-x}$

(B) $\frac{d^3y}{dx^3} = \frac{dy}{dx}$

(C) $f^{(4)}(x) = f'(x) \cdot f'''(x)$

(D) $y - \frac{d^4y}{dx^4} = 3x$

$(\sin x + e^{-x} + 3x) - (\sin x + e^{-x}) = 3x$

$y = \sin x + e^{-x} + 3x$

$y' = \cos x - e^{-x} + 3$

$y'' = -\sin x + e^{-x}$

$y^{3}(x) = -\cos x - e^{-x}$

$y^{4}(x) = \sin x + e^{-x}$