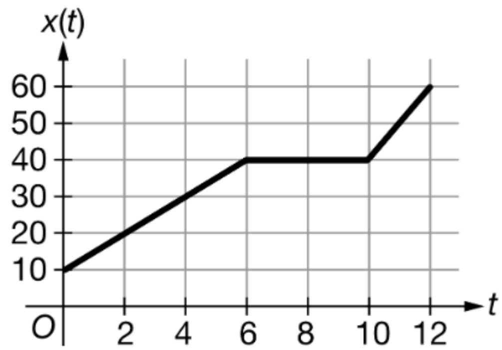
Unit 3 AP Classroom Practice for Sections 1-5

1)



A particle is moving on the x-axis and the position of the particle at time t is given by $x(t)$, whose graph is shown above. Which of the following is the best estimate for the speed of the particle at time $t = 4$?

(A) 0

(B) 5

(C) $\frac{15}{2}$

(D) 10

2)

x	-2	-1	0	1	2
$g(x)$	-3	2	1	0	5

Selected values of a function g are shown in the table above. What is the average rate of change of g over the interval $[-2, 2]$?

(A) $\frac{2-(-2)}{5-(-3)}$

(B) $\frac{5-(-3)}{2-(-2)}$

(C) $\frac{5+(-3)}{2}$

(D) $\frac{-3+2+1+0+5}{5}$

3) A car is driven on a straight road, and the distance traveled by the car after time $t = 0$ is given by a quadratic function s, where $s(t)$ is measured in feet and t is measured in seconds for $0 \leq t \leq 12$. Of the following, which gives the best estimate of the velocity of the car, in feet per second, at time $t = 6$ seconds?

(A) $s(6)$

(B) $\frac{s(6)}{6}$

(C) $\frac{s(8)-s(4)}{8-4}$

(D) $\frac{s(7)-s(5)}{7-5}$

4) Let f be the function defined by $f(x) = e^{2x}$. The average rate of change of f over the interval $[1, b]$ is 20, where $b > 1$. Which of the following is an equation that could be used to find the value of b?

(A) $f(b) = 20$

(B) $f(b) - f(1) = 20$

(C) $\frac{f(b)-f(1)}{b-1} = 20$

(D) $\frac{f(b)+f(1)}{2} = 20$

5) The function $t = f(S)$ models the time, in hours, for a sample of water to evaporate as a function of the size S of the sample, measured in milliliters. What are the units for $f''(S)$?

(A) hours per milliliter

(B) milliliters per hour

(C) hours per milliliter per milliliter

(D) milliliters per hour per hour

6) Graphing Calculator Needed

$$D(t) = 10 + 4.9\cos\left(\frac{\pi}{6}t\right)$$

The function D defined above models the depth, in feet, of the water t hours after 12 A.M. in a certain harbor. Which of the following presents the method for finding the instantaneous rate of change of the depth of the water, in feet per hour, at 6 A.M. ?

(A) $\frac{D(6)-D(0)}{6-0} = -1.633$

(B) $D'(6) = 0$

(C) $D''(6) = 1.343$

(D) $D(6) = 5.100$

7) A particle moves along the x-axis so that at any time $t \geq 0$ its position is given by $x(t) = \frac{1}{2}(a-t)^2$, where a is a positive constant. For what values of t is the particle moving to the right?

(A) The particle is moving to the right only if $0 < t < a$.

(B) The particle is moving to the right only if $a < t$.

(C) The particle is moving to the right only if $t = a$.

(D) The particle is not moving to the right.

8) An object moves along a straight line so that at any time t, $0 \leq t \leq 9$, its position is given by $x(t) = 7 + 6t - t^2$. For what value of t is the object at rest?

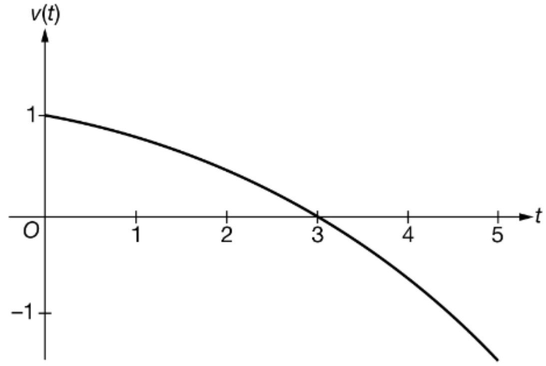
(A) $t = 3$

(B) $t = 6$

(C) $t = \frac{13}{2}$

(D) $t = 7$

9)



A particle traveling on the x-axis has position $x(t)$ at time t. The graph of the particle's velocity $v(t)$ is shown above for $0 \leq t \leq 5$. Which of the following expressions gives the total distance traveled by the particle over the time interval $0 \leq t \leq 5$?

(A) $x(0) - x(5)$

(B) $x(5) - x(0)$

(C) $(x(3) - x(0)) + (x(3) - x(5))$

(D) $(x(0) - x(3)) + (x(5) - x(3))$

10) Let x and y be functions of time t such that the sum of x and twice y is constant. Which of the following equations describes the relationship between the rate of change of x with respect to time and the rate of change of y with respect to time?

(A) $\frac{dx}{dt} = 2\frac{dy}{dt}$

(B) $\frac{dx}{dt} = -2\frac{dy}{dt}$

(C) $2\frac{dx}{dt} + \frac{dy}{dt} = 0$

(D) $\frac{dx}{dt} + 2\frac{dy}{dt} = K$, where K is a function of t

11) A right triangle has base x feet and height h feet, where x is constant and h changes with respect to time t, measured in seconds. The angle θ, measured in radians, is defined by $\tan\theta = \frac{h}{x}$. Which of the following best describes the relationship between $\frac{d\theta}{dt}$, the rate of change of θ with respect to time, and $\frac{dh}{dt}$, the rate of change of h with respect to time?

(A) $\frac{d\theta}{dt} = \left(\frac{x}{x^2+h^2}\right)\frac{dh}{dt}$ radians per second

(B) $\frac{d\theta}{dt} = \left(\frac{x^2}{x^2+h^2}\right)\frac{dh}{dt}$ radians per second

(C) $\frac{d\theta}{dt} = \left(\frac{1}{\sqrt{x^2+h^2}}\right)\frac{dh}{dt}$ radians per second

(D) $\frac{d\theta}{dt} = \tan^{-1}\left(\frac{1}{x}\frac{dh}{dt}\right)$ radians per second

12) A particle moves on the hyperbola $xy = 15$ for time $t \geq 0$ seconds. At a certain instant, $x = 3$ and $\frac{dx}{dt} = 6$. Which of the following is true about y at this instant?

(A) y is decreasing by 10 units per second.

(B) y is increasing by 10 units per second.

(C) y is decreasing by 5 units per second.

(D) y is increasing by 5 units per second.

13) A piece of rubber tubing maintains a cylindrical shape as it is stretched. At the instant that the inner radius of the tube is 2 millimeters and the height is 20 millimeters, the inner radius is decreasing at the rate of 0.1 millimeter per second and the height is increasing at the rate of 3 millimeters per second. Which of the following statements about the volume of the tube is true at this instant? (The volume V of a cylinder with radius r and height h is $V = \pi r^2 h$.)

(A) The volume is increasing by 4π cubic millimeters per second.

(B) The volume is decreasing by 4π cubic millimeters per second.

(C) The volume is increasing by 20π cubic millimeters per second.

(D) The volume is decreasing by 20π cubic millimeters per second.

14)

x	1	3	5	7	9
$f(x)$	0	6	18	29	42

Selected values of a differentiable function f are given in the table above. What is the fewest possible number of values of c in the interval $[1, 9]$ for which the Mean Value Theorem guarantees that $f'(c) = 6$?

(A) Zero

(B) One

(C) Two

(D) Three

15) The Mean Value Theorem can be applied to which of the following functions on the closed interval $[-5, 5]$?

(A) $f(x) = \frac{1}{\sin x}$

(B) $f(x) = \frac{x-1}{|x-1|}$

(C) $f(x) = \frac{x^2}{x^2-36}$

(D) $f(x) = \frac{x^2}{x^2-4}$

16) Which of the following functions of x is guaranteed by the Extreme Value Theorem to have an absolute maximum on the interval $[0, 2\pi]$?

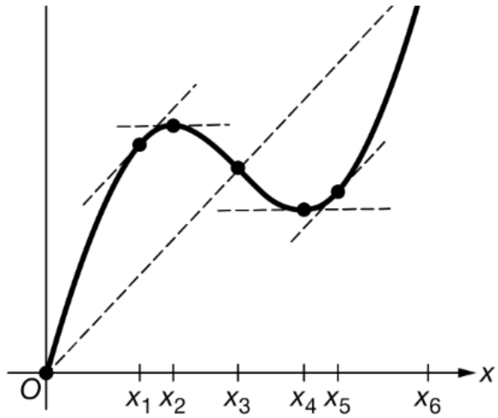
(A) $y = \frac{1}{1+\sin x}$

(B) $y = \frac{1}{x^2+\pi}$

(C) $y = \frac{x^2-2\pi x+\pi^2}{x-\pi}$

(D) $y = \frac{|x-\pi|}{x-\pi}$

17)



Graph of g

The function g shown in the figure above is continuous on the closed interval $[0, x_6]$ and differentiable on the open interval $(0, x_6)$, where x_1, x_2, x_3, x_4, x_5, and x_6 are points on the x-axis. Based on the graph, what are all values of x that satisfy the conclusion of the Mean Value Theorem applied to g on the closed interval $[0, x_6]$?

(A) x_3 only, because this is the value where $g(x)$ equals the average rate of change of g on $[0, x_6]$.

(B) x_2 and x_4 only, because these are the values where $g'(x) = 0$ on $[0, x_6]$.

(C) x_1 and x_5 only, because these are the values where the instantaneous rate of change of g at those values is equal to the average rate of change of g on $[0, x_6]$.

(D) x_1, x_3, and x_5 only, because these are the values where either the instantaneous rate of change of g at the value is equal to the average rate of change of g on $[0, x_6]$ or the value of $g(x)$ is equal to the average rate of change of g on $[0, x_6]$.

18)

x	0	1	2	3
$f(x)$	0	4	7	6

Let f be a function with selected values given in the table above. Which of the following statements must be true?

I. By the Intermediate Value Theorem, there is a value c in the interval $(0, 3)$ such that $f(c) = 2$.

II. By the Mean Value Theorem, there is a value c in the interval $(0, 3)$ such that $f'(c) = 2$.

III. By the Extreme Value Theorem, there is a value c in the interval $[0, 3]$ such that $f(c) \leq f(x)$ for all x in the interval $[0, 3]$.

(A) None

(B) I only

(C) II only

(D) I, II, and III

19) Let f be the function defined by $f(x) = (\sin x)e^{-x}$ on the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$. On which of the following open intervals is f increasing?

(A) $\left(-\frac{\pi}{4}, \frac{\pi}{2}\right)$

(B) $\left(0, \frac{\pi}{2}\right)$ only

(C) $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$ only

(D) $\left(-\frac{\pi}{2}, \frac{\pi}{4}\right)$

20) Graphing Calculator Needed

Let f be the function with derivative given by $f'(x) = \sin x + \cos(2x) - \frac{\pi}{4}$ for $0 \leq x \leq \pi$. On which of the following intervals is f increasing?

(A) $[0, 0.724]$ only

(B) $[0, 0.724]$ and $[2.418, 3.142]$

(C) $[0, 0.253]$ and $[1.571, 2.889]$

(D) $[0.724, 2.418]$

21) Let f be the function with derivative given by $f'(x) = x^2 - a^2 = (x-a)(x+a)$, where a is a positive constant. Which of the following statements is true?

(A) f is decreasing for $-a < x < a$ because $f'(x) < 0$ for $-a < x < a$.

(B) f is decreasing for $x < -a$ and $x > a$ because $f'(x) < 0$ for $x < -a$ and $x > a$.

(C) f is decreasing for $x < 0$ because $f'(x) < 0$ for $x < 0$.

(D) f is decreasing for $x < 0$ because $f''(x) < 0$ for $x < 0$.

22) The function f is defined by $f(x) = x^2e^{-x^2}$. At what values of x does f have a relative maximum?

(A) -2

(B) 0

(C) 1 only

(D) -1 and 1

23) Let f be a differentiable function with a domain of $(0, 10)$. It is known that $f'(x)$, the derivative of $f(x)$, is negative on the intervals $(0, 2)$ and $(4, 6)$ and positive on the intervals $(2, 4)$ and $(6, 10)$. Which of the following statements is true?

(A) f has no relative minima and three relative maxima.

(B) f has one relative minimum and two relative maxima.

(C) f has two relative minima and one relative maximum.

(D) f has three relative minima and no relative maxima.

24) Graphing Calculator Needed

Let f be the function with derivative $f'(x) = x^3 - 3x - 2$. Which of the following statements is true?

(A) f has no relative minima and one relative maximum.

(B) f has one relative minimum and no relative maxima.

(C) f has one relative minimum and one relative maximum.

(D) f has two relative minima and one relative maximum.

25)

Let g be the function defined by $g(x) = |x| - 3|x + 1|$. What is the absolute maximum value of g on the closed interval $[-2, 2]$?

(A) 1

(B) -1

(C) -3

(D) -7

26) Let f be the function defined by $f(x) = 3x^2 - x^3$. What is the absolute minimum value of f on the closed interval $\left[1, \frac{5}{2}\right]$?

(A) 0

(B) 2

(C) $\frac{25}{8}$

(D) 4

27) At what values of x does the graph of $y = x^2 e^{-2x}$ have a point of inflection?

(A) $x = -2$ and $x = 0$

(B) $x = 0$ and $x = 1$

(C) $x = -2 - \sqrt{2}$ and $x = -2 + \sqrt{2}$

(D) $x = 1 - \frac{\sqrt{2}}{2}$ and $x = 1 + \frac{\sqrt{2}}{2}$

28) Graphing Calculator Needed

The second derivative of the function g is given by $g''(x) = x^5 - 2.2x^4 - 6.61x^3 + 8.602x^2$. At which values of x in the interval $-3 < x < 4$ does the graph of g have a point of inflection where the concavity of the graph changes from concave up to concave down?

(A) $x = 1.1$ only

(B) $x = -2.3$ and $x = 3.4$ only

(C) $x = -2.3$, $x = 1.1$, and $x = 3.4$ only

(D) $x = -2.3$, $x = 0$, $x = 1.1$, and $x = 3.4$

29) Graphing Calculator Needed

The first derivative of the function h is given by $h'(x) = x^5 - 3x^2 + x$. What are all intervals on which the graph of h is concave down?

(A) $(-\infty, 0)$ and $(0.338, 1.307)$

(B) $(-\infty, 0.669)$

(C) $(-\infty, 0.167)$ and $(1, \infty)$

(D) $(0.167, 1)$

30) Let f be a function such that $f(-1) = 1$. At each point (x, y) on the graph of f, the slope is given by $\frac{dy}{dx} = -x^2 - xy + y^2 - 1$. Which of the following statements is true?

(A) f has a relative minimum at $x = -1$.

(B) f has a relative maximum at $x = -1$.

(C) f has neither a relative minimum nor a relative maximum at $x = -1$.

(D) There is insufficient information to determine whether f has a relative minimum, a relative maximum, or neither at $x = -1$.

31) Let f be a twice-differentiable function. Which of the following statements are individually sufficient to conclude that $x = 2$ is the location of the absolute maximum of f on the interval $[-5, 5]$?

I. $f'(2) = 0$

II. $x = 2$ is the only critical point of f on the interval $[-5, 5]$, and $f''(2) < 0$.

III. $x = 2$ is the only critical point of f on the interval $[-5, 5]$, and $f(-5) < f(5) < f(2)$.

(A) II only

(B) III only

(C) I and II only

(D) II and III only

32)

x	0	1	2	3	4	5
$f'(x)$	-3	0	-1	5	0	-3
$f''(x)$	5.3	-2.0	1.7	-0.5	1.2	-5.1

Let f be a twice-differentiable function. Selected values of f' and f'' are shown in the table above. Which of the following statements are true?

I. f has neither a relative minimum nor a relative maximum at $x = 1$.

II. f has a relative maximum at $x = 1$.

III. f has a relative maximum at $x = 4$.

(A) I only

(B) II only

(C) III only

(D) I and III only

33) Let f be the function defined by $f(x) = \frac{1}{3}x^3 - 3x^2 - 16x$. On which of the following intervals is the graph of f both decreasing and concave down?

(A) $(-\infty, 3)$

(B) $(-2, 3)$ only

(C) $(3, 8)$

(D) $(8, \infty)$

34)

x	$0 < x < 3$	$x = 3$	$3 < x < 9$	$x = 9$	$9 < x < 11$	$x = 11$	$11 < x < 16$
$f'(x)$	Positive	Undefined	Negative	-3	Negative	0	Positive
$f''(x)$	Positive	Undefined	Negative	0	Positive	0	Positive

The function f is continuous on the interval $(0, 16)$, and f is twice differentiable except at $x = 3$, where the derivatives are undefined. Information about the first and second derivatives of f for values of x in the interval $(0, 16)$ is given in the table above. At what values of x in the interval $(0, 16)$ does the graph of f have a point of inflection?

(A) $x = 9$ only

(B) $x = 3$ and $x = 9$

(C) $x = 3$ and $x = 11$

(D) $x = 9$ and $x = 11$

35) Graphing Calculator Needed

The first derivative of the function h is given by $h'(x) = \sin x + \cos(x^2) + x$, and the second derivative of h is given by $h''(x) = \cos x - 2x\sin(x^2) + 1$. On what open intervals contained in $-3 < x < 2$ is the graph of h both increasing and concave down?

(A) $(0.969, 1.697)$ only

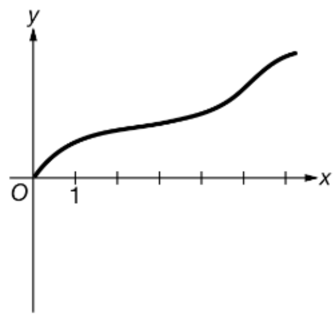
(B) $(-2.499, -1.829)$ and $(0.969, 1.697)$

(C) $(-0.495, 2)$

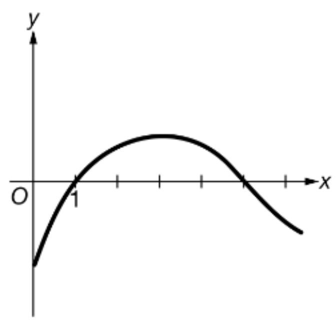
(D) $(-1.311, -0.166)$

36) The function f is differentiable and increasing on the interval $0 \le x \le 6$, and the graph of f has exactly two points of inflection on this interval. Which of the following could be the graph of f', the derivative of f ?

(A)



(B)



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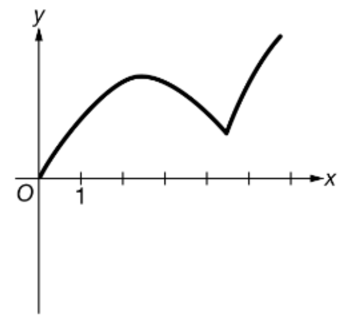

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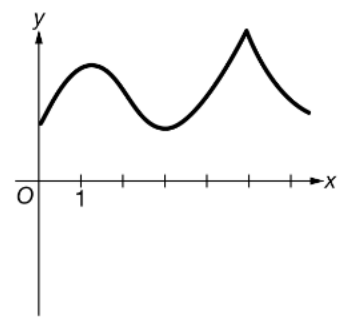

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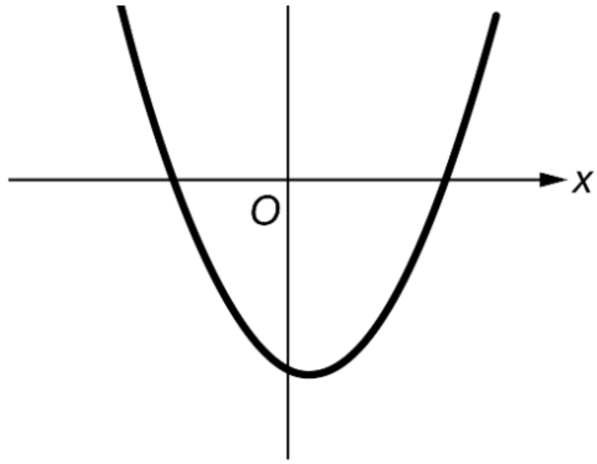
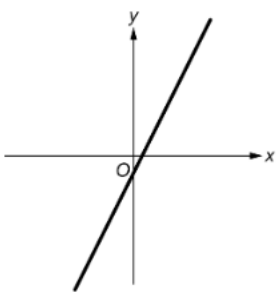


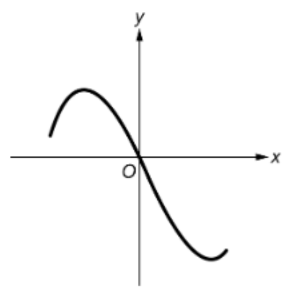
Graph of f'

The graph of f', the derivative of the function f, is shown above. Which of the following could be the graph of f?

(A)



(C)



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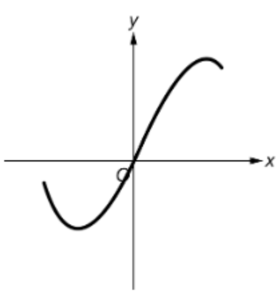
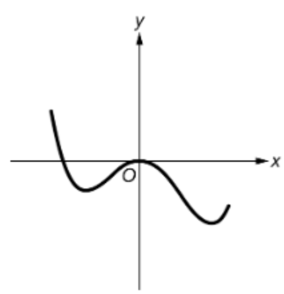
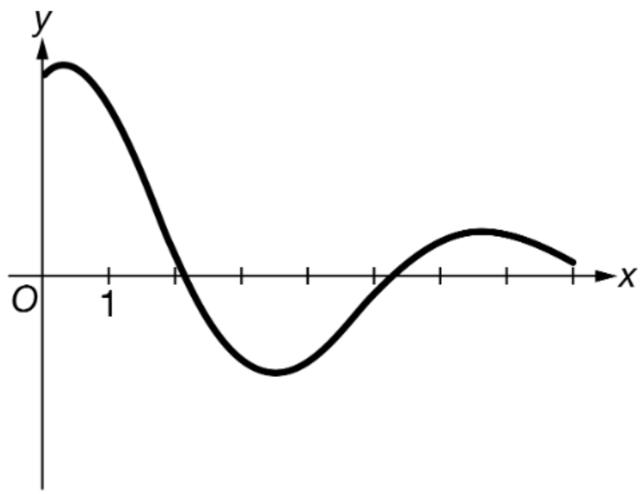


(D)



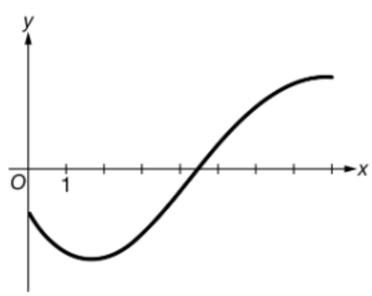
38)



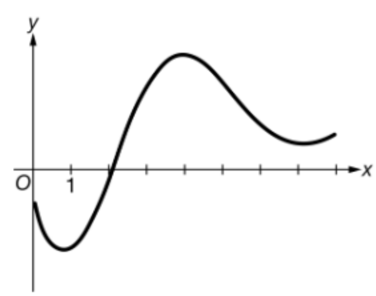
Graph of f''

The graph of f'', the second derivative of the function f, is shown above on the interval $0 \leq x \leq 8$. Which of the following could be the graph of f ?

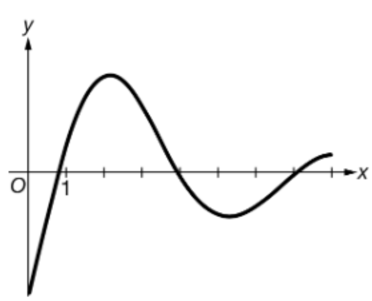
(A)



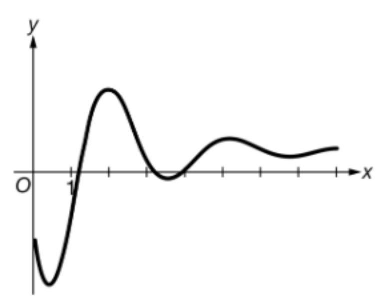
(C)



(B)



(D)



39) Let C be the curve defined by $x^2y = 4$. Which of the following statements is true of curve C at the point $(2, 1)$?

(A) It has a relative minimum because $y' = 0$ and $y'' > 0$.

(B) It has a relative maximum because $y' = 0$ and $y'' < 0$.

(C) It is decreasing and concave up because $y' < 0$ and $y'' > 0$.

(D) It is decreasing and concave down because $y' < 0$ and $y'' < 0$.

40) Consider the curve defined by $\frac{x^2}{16} - \frac{y^2}{9} = 1$. It is known that $\frac{dy}{dx} = \frac{9x}{16y}$ and $\frac{d^2y}{dx^2} = -\frac{81}{16y^3}$. Which of the following statements is true about the curve in Quadrant IV ?

(A) The curve is concave up because $\frac{dy}{dx} > 0$.

(B) The curve is concave down because $\frac{dy}{dx} < 0$.

(C) The curve is concave up because $\frac{d^2y}{dx^2} > 0$.

(D) The curve is concave down because $\frac{d^2y}{dx^2} < 0$.