# Unit 3 AP Classroom Practice for Sections 6-9

1) Let $f$ and $g$ be inverse functions that are differentiable for all $x$. If $f(3) = -2$ and $g'(-2) = -4$, which of the following statements must be false?

$f$ $(3,-2)$ $g$ $(-2,3)$ $g'(-2) = -4$

so... $\boxed{f'(3) = \frac{-1}{4}}$

I. $f'(0) = \frac{1}{4}$

II. $f'(3) = -\frac{1}{4}$

III. $f'(5) = -\frac{1}{4}$

(A) I only

(B) II only

(C) III only

(D) I and III only

$f'(3) = \frac{-1}{4}$ so... all $f'(x)$ values MUST BE NEGATIVE OTHERWISE THE INVERSE COULD NOT EXIST

ANY function that inc/dec or dec/inc FAILS HLT

2) For which of the following decreasing functions $f$ does $(f^{-1})'(10) = -\frac{1}{8}$?

so $x$-coord on $f^{-1}$ and $y$-coord on $f$

(A) $f(x) = -5x + 15$ $\quad f'(x) = -5$ $\quad -8 \neq -5$

(B) $f(x) = -2x^3 - 2x + 14$ $\quad f'(x) = -6x^2 - 2$

(C) $f(x) = -x^5 - 4x + 15$ $\quad f'(x) = -5x^4 - 4$

(D) $f(x) = e^{-2x} - x + 9$ $\quad f'(x) = -2e^{-2x} - 1$

$-6x^2 - 2 = -8$
$x^2 = 1$
$x = \pm 1$
Is $\boxed{(1,10)}$ or $(-1, \cancel{10})$ on $f(x)$? yes!

$-5x^4 - 4 = -8$
$x^4 = \frac{4}{5}$
$x = \pm\sqrt[4]{\frac{4}{5}}$
$(\sqrt[4]{4/5}, 10)$, $(\sqrt[4]{4/5}, -10)$ cannot be on $f(x) = -x^5 - 4x + 15$

$-2e^{-2x} - 1 = -8$
$e^{-2x} = \frac{7}{2}$
$x = \frac{\ln(\frac{7}{2})}{-2}$
Is $\cancel{\left(\frac{\ln(7/2)}{-2}, 10\right)}$ on $f(x)$? NO!

3) An increasing function $f$ satisfies $f(10) = 5$ and $f'(10) = 8$. Which of the following statements about the inverse of $f$ must be true?

$f \; (10,5)$
$f'(10) = 8$

$f^{-1}(5,10)$
$(f^{-1})'(5) = \frac{1}{8}$

(A) $(f^{-1})'(5) = 10$

(B) $(f^{-1})'(8) = 10$

(C) $(f^{-1})'(5) = 8$

(D) $(f^{-1})'(5) = \frac{1}{8}$

Tough! 4)

$g(x) \to (2,-5)$
$g^{-1}(x) \to (-5,2)$

| $x$ | 4 | 8 |
|---|---|---|
| | | |
| $f(x)$ | 11 | 6 |
| $f'(x)$ | $-4$ | $-3$ |

$g(x) = f(4x) - f(2x)$
$g'(x) = f'(4x)\cdot 4 - f'(2x)\cdot 2$
$g'(2) = \underbrace{f'(8)\cdot 4 - f'(4)\cdot 2}_{\text{slope of } g(x) \text{ at } (2,-5)}$

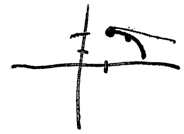
The table above gives selected values for a differentiable and decreasing function $f$ and its derivative. Let $g$ be the decreasing function given by $g(x) = f(4x) - f(2x)$, where $g(2) = f(8) - f(4) = -5$. Which of the following describes a correct process for finding $(g^{-1})'(-5)$? $= \frac{1}{f'(8)\cdot 4 - f'(4)\cdot 2} = \frac{1}{(-3)\cdot 4 - (-4)\cdot 2} = \frac{1}{-12+8} = -\frac{1}{4}$

(A) $(g^{-1})'(-5) = \frac{1}{g'(g^{-1}(-5))} = \frac{1}{g'(2)}$ and $g'(2) = 4f'(8) - 2f'(4)$

(B) $(g^{-1})'(-5) = \frac{1}{g'(g^{-1}(-5))} = \frac{1}{g'(2)}$ and $g'(2) = f'(8) - f'(4)$

(C) $(g^{-1})'(-5) = g'(g^{-1}(-5)) = g'(2)$ and $g'(2) = f'(8) - f'(4)$

(D) $(g^{-1})'(-5) = g'(g^{-1}(-5)) = g'(2)$ and $g'(2) = 4f'(8) - 2f'(4)$



$\frac{dy}{dx}(1,2) = 4-1$
$= 3$

5) Let $f$ be a function such that at each point $(x, y)$ on the graph of $f$, the slope is given by $\frac{dy}{dx} = y^2 - x$.

so tangent line above graph

The graph of $f$ passes through the point $(1, 2)$ and is concave down on the interval $1 < x < 1.5$. Let $k$ be the approximation for $f(1.2)$ found by using the locally linear approximation of $f$ at $x = 1$. Which of the following statements about $k$ is true?

$y = 3(x-1)+2$
$y(1.2) = 3(.2)+2$
$= 2.6$

(A) $k = 5.6$ and is an overestimate for $f(1.2)$.

(B) $k = 5.6$ and is an underestimate for $f(1.2)$.

tangent line above graph of f

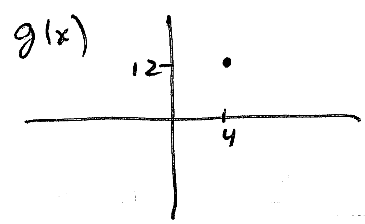
(C) $k = 2.6$ and is an overestimate for $f(1.2)$.

(D) $k = 2.6$ and is an underestimate for $f(1.2)$.

6) $g'(4) = 2.2$

$y = 2.2(x-4)+12$
$y(4.2) = 2.2(4.2-4)+12$
$= 2.2 \cdot (.2)+12$
$= .44+12$

| $x$ | 3.8 | 4.0 | 4.2 | 4.4 |
|---|---|---|---|---|
| $g'(x)$ | −0.8 | 2.2 | 1.8 | −1.2 |



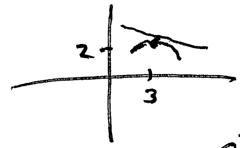

Selected values of the derivative of the function $g$ are given in the table above. It is known that $g(4) = 12$. What is the approximation for $g(4.2)$ found using the line tangent to the graph of $g$ at $x = 4$?

(A) 12.44

(B) 12.40

(C) 12.36

(D) 11.60




tan line above $g(x)$

7) Let $g$ be a differentiable function such that $g(3) = 2$ and $g'(3) = -\frac{3}{4}$. The graph of $g$ is concave down on the interval $(2, 4)$. Which of the following is true about the approximation for $g(2.6)$ found using the line tangent to the graph of $g$ at $x = 3$?

$y = \frac{-3}{4}(x-3) + 2$

$y(2.6) = \frac{-3}{4}(2.6-3) + 2 = \frac{-3}{4}\left(\frac{-4}{10}\right) + 2 = \frac{3}{10} + 2$

(A) $g(2.6) \approx 1.7$ and this approximation is an overestimate of the value of $g(2.6)$.

(B) $g(2.6) \approx 1.7$ and this approximation is an underestimate of the value of $g(2.6)$.

tan line above function

(C) $g(2.6) \approx 2.3$ and this approximation is an overestimate of the value of $g(2.6)$.

(D) $g(2.6) \approx 2.3$ and this approximation is an underestimate of the value of $g(2.6)$.

8) Let $f$ be the function defined by $f(x) = 3x + 2e^{-3x}$, and let $g$ be a differentiable function with derivative given by $g'(x) = 4 + \frac{1}{x}$. It is known that $\lim\limits_{x\to\infty} g(x) = \infty$. The value of $\lim\limits_{x\to\infty} \frac{f(x)}{g(x)}$ is

(A) 0

(B) $\frac{3}{4}$

(C) 1

(D) nonexistent

$\lim\limits_{x\to\infty} \frac{f(x)}{g(x)} = \frac{\infty}{\infty}$

so ... $= \lim\limits_{x\to\infty} \frac{f'(x)}{g'(x)}$

$= \lim\limits_{x\to\infty} \frac{3-6e^{-3x}}{4+\frac{1}{x}} = \frac{3-6\cdot 0}{4+0}$

$= \frac{3}{4}$

9) $\lim_{t\to 0}\frac{\sin t}{\ln(2e^t-1)} = \frac{\sin(0)}{\ln(2e^0-1)} = \frac{0}{\ln 1} = \frac{0}{0}$

Ⓐ $-1$

Ⓑ 0

Ⓒ $\frac{1}{2}$

Ⓓ 1

$$= \lim_{t\to 0}\frac{\cos t}{\frac{1}{2e^t-1}\cdot 2e^t} = \frac{\cos(0)}{\frac{1}{2e^0-1}\cdot 2e^0} = \frac{1}{\frac{1}{2-1}\cdot 2} = \frac{1}{1\cdot 2} = \frac{1}{2}$$

10) $\lim_{x\to\frac{\pi}{2}}\frac{3\cos x}{2x-\pi}$ is $= \frac{3\cos\frac{\pi}{2}}{2\cdot\frac{\pi}{2}-\pi} = \frac{3\cdot 0}{\pi-\pi} = \frac{0}{0}$

Ⓐ $-\frac{3}{2}$

Ⓑ 0

Ⓒ $\frac{3}{2}$

Ⓓ nonexistent

$$= \lim_{x\to\frac{\pi}{2}}\frac{-3\sin x}{2}$$

$$= \frac{-3\cdot\sin(\frac{\pi}{2})}{2}$$

$$= \frac{-3\cdot 1}{2} = \frac{-3}{2}$$

11) $\lim_{x \to \pi} \frac{x+\pi \sec x}{x^2-\pi^2}$ is $= \frac{\pi + \pi \cdot \sec(\pi)}{\pi^2 - \pi^2} = \frac{\pi + \pi(-1)}{\pi^2 - \pi^2} = \frac{0}{0}$

Ⓐ $-\frac{\pi}{2}$

Ⓑ 0

Ⓒ $\frac{1}{2\pi}$

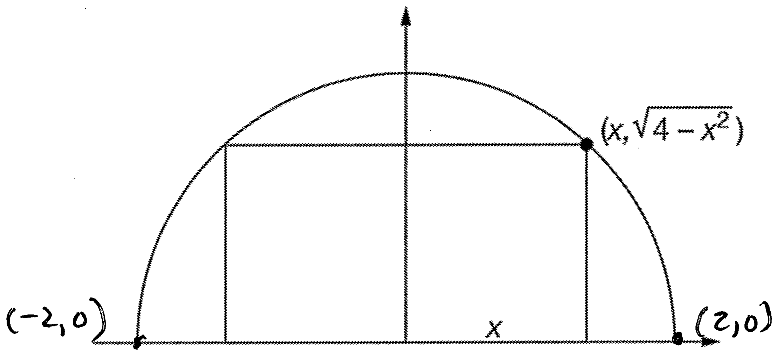
Ⓓ nonexistent

$$= \lim_{x \to \pi} \frac{1 + \pi \sec x \tan x}{2x} = \frac{1 + \pi \cdot \sec\pi \tan\pi}{2(\pi)}$$

$$= \frac{1 + \pi \cdot (-1)(0)}{2\pi}$$

$$= \frac{1}{2\pi}$$

12)




$A(x) = 2xy$

The figure above shows a rectangle inscribed in a semicircle with a radius of 2. The area of such a rectangle is given by $A(x) = 2x\sqrt{4-x^2}$, where the width of the rectangle is $2x$. It can be shown that $A'(x) = \frac{-2x^2}{\sqrt{4-x^2}} + 2\sqrt{4-x^2}$ and $A$ has critical values of $-2, -\sqrt{2}, \sqrt{2}$, and 2. It can also be shown that $A'(x)$ changes from positive to negative at $x = \sqrt{2}$. Which of the following statements is true?

(A) The inscribed rectangle with maximum area has dimensions $\sqrt{2}$ by $\sqrt{2}$.

(B) The inscribed rectangle with minimum area has dimensions $\sqrt{2}$ by $\sqrt{2}$.

(C) The inscribed rectangle with maximum area has dimensions $2\sqrt{2}$ by $\sqrt{2}$.

(D) The inscribed rectangle with minimum area has dimensions $2\sqrt{2}$ by $\sqrt{2}$.

$-\quad+\quad-$

$-2 \quad -\sqrt{2} \quad \sqrt{2} \quad 2$

rel MAX when $x = \sqrt{2}$

$A'(x) = \frac{-2x^2 + 2(4-x^2)}{\sqrt{4-x^2}}$

$A'(x) = \frac{8-4x^2}{\sqrt{4-x^2}}$

from ordered pair in diagram $\rightarrow$ $f(\sqrt{2}) = \sqrt{4-(\sqrt{2})^2}$

$= \sqrt{4-2}$

$= \sqrt{2}$

$L = 2x$

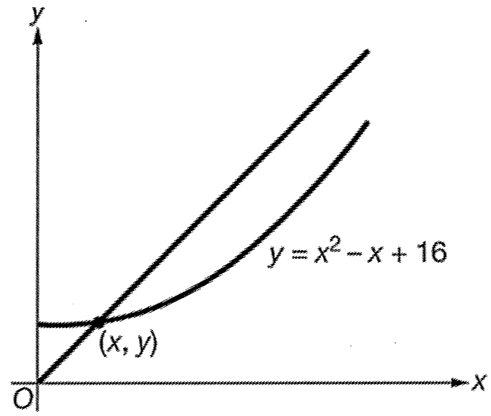
$W = y$

$L = 2\sqrt{2}$

$W = \sqrt{2}$

13)

Looking to min/max slope of the line




(0,0)

Consider all lines in the ~~$xy$-plane~~ that pass through both the origin and a point $(x, y)$ on the graph of $y = x^2 - x + 16$ for $1 \le x \le 8$. The figure above shows ~~one such~~ line and the graph of $y = x^2 - x + 16$. Which of the following statements is true?

(A) The line with minimum slope passes through the graph of $y = x^2 - x + 16$ at $x = 1$.

(B) The line with minimum slope passes through the graph of $y = x^2 - x + 16$ at $x = 4$.

(C) The line with minimum slope passes through the graph of $y = x^2 - x + 16$ at $x = 7$.

(D) The line with minimum slope passes through the graph of $y = x^2 - x + 16$ at $x = 8$.

$$m = \frac{y-0}{x-0}$$

$$m = \frac{x^2 - x + 16}{x}$$

$$m = x - 1 + 16x^{-1}$$

$$m'(x) = 1 - 16x^{-2}$$

$$0 = 1 - \frac{16}{x^2}$$

$$0 = x^2 - 16$$

$$0 = (x+4)(x-4)$$

$x = -4$ or $x = 4$

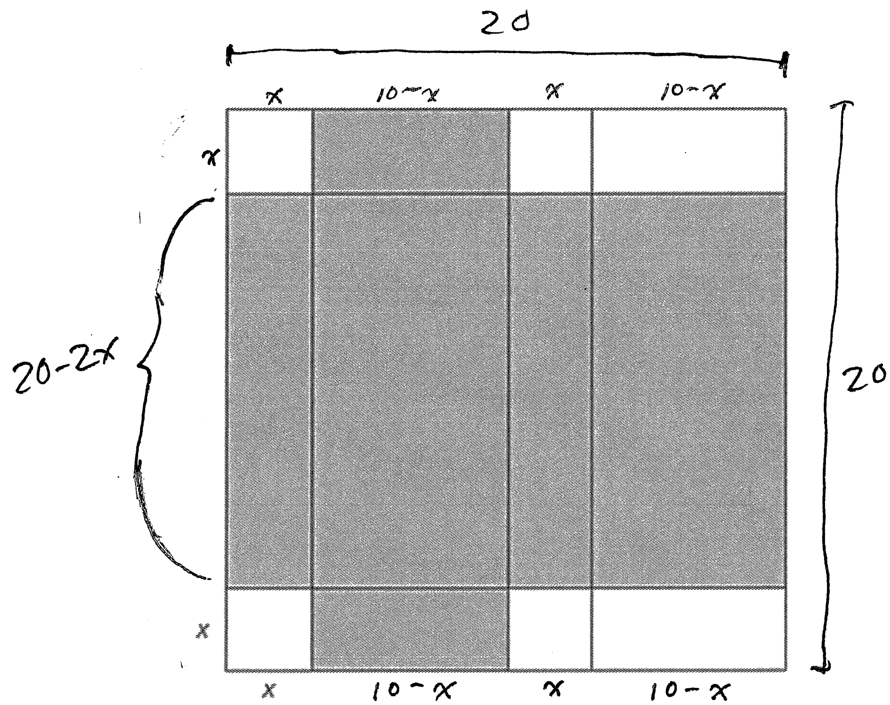
$m'$   $-4$, $0$   $-$   $4$   $+$

$1 \le x \le 8$

$$m'(x) = \frac{x^2 - 16}{x^2}$$

rel min for slope when $x = 4$

14)




The figure above represents a square sheet of cardboard with side length 20 inches. The sheet is cut and pieces are discarded. When the cardboard is folded, it becomes a rectangular box with a lid. The pattern for the rectangular box with a lid is shaded in the figure. Four squares with side length $x$ and two rectangular regions are discarded from the cardboard. Which of the following statements is true? (The volume $V$ of a rectangular box is given by $V = lwh$.)

(A) When $x = 10$ inches, the box has a minimum possible volume.

(B) When $x = 10$ inches, the box has a maximum possible volume.

(C) When $x = \frac{10}{3}$ inches, the box has a minimum possible volume.

(D) When $x = \frac{10}{3}$ inches, the box has a maximum possible volume.

$V = l \cdot w \cdot h$

$V = (20-2x)(10-x)(x)$

$V = (200-40x+2x^2)\cdot(x)$

$V = 2x^3-40x^2+200x$

$V'(x) = 6x^2-80x+200$

$V'(x) = 2(3x^2-40x+100)$

$V'(x) = 2(3x-10)(x-10)$

$0 = 2(3x-10)(x-10)$

$x = \frac{10}{3}$ or $x = 10$

$+ \quad - \quad +$

$0 \quad \frac{10}{3} \quad 10$

MAX volume

MIN volume