Unit 5 Quiz Open Ended Questions

6

Name Key

Let f be a function with $f(0)=\frac{1}{2}$ such that all points (x, y) on the graph of f satisfy the differential equation $\frac{dy}{dx}=\frac{y}{2}\left(1-\frac{x}{32}\right)$. Find $y=f(x)$.

$$\int \frac{1}{y}\,dy = \int \frac{1}{2} - \frac{1}{64}x \, dx$$

$$\ln|y| = \frac{1}{2}x - \frac{1}{128}x^2 + C$$

$$\ln\left(\frac{1}{2}\right) = 0 - 0 + C$$

$$\ln\left(\frac{1}{2}\right) = C$$

or

$-\ln 2$

$$\ln|y| = \frac{1}{2}x - \frac{1}{128}x^2 + \ln\left(\frac{1}{2}\right)$$

OR $(-\ln 2)$

$$e^{\frac{1}{2}x - \frac{1}{128}x^2 + \ln\left(\frac{1}{2}\right)} = |y|$$

$$\pm e^{\frac{1}{2}x - \frac{1}{128}x^2 + \ln\left(\frac{1}{2}\right)} = y$$

$$e^{\frac{1}{2}x - \frac{1}{128}x^2 + \ln\left(\frac{1}{2}\right)} = y$$

or

$$e^{\frac{1}{2}x - \frac{1}{128}x^2} \cdot e^{\ln\left(\frac{1}{2}\right)} = y$$

$$\frac{1}{2}e^{\frac{1}{2}x - \frac{1}{128}x^2} = y$$

Unit 5 Quiz Open Ended Questions

6

Name Key

If $\frac{dy}{dx} = (y-4)\sec^2 x$ and $y(0) = 5$, then $y =$

$$\frac{1}{y-4}\,dy = \sec^2 x\,dx$$

$$\int \frac{1}{y-4}\,dy = \int \sec^2 x\,dx$$

$u = y-4$

$\frac{du}{dy} = 1$

$$\int \frac{1}{u}\,du = \tan x + C$$

$$\ln|u| = \tan x + C$$

(4) $$\ln|y-4| = \tan x + C$$

$$\ln|5-4| = \tan(0) + C$$

$$\ln|1| = 0 + C$$

$$\ln 1 = C$$

$$0 = C$$

$$\ln|y-4| = \tan x$$

$$e^{\tan x} = |y-4|$$

(2) $$\pm e^{\tan x} = y-4$$

$$4 \pm e^{\tan x} = y$$

$$\boxed{y = 4 + e^{\tan x}}$$

Unit 5 Quiz Open Ended Questions 6 Name Key

Find the particular solution $y = f(x)$ to the differential equation $\frac{dy}{dx} = \frac{2x+3}{ye^{(y^2+1)}}$

with the initial condition $y(0) = -1$.

$$y e^{y^2+1} dy = 2x+3\,dx$$

$$\int y e^{y^2+1} dy = \int 2x+3\,dx$$

$u = y^2+1$

$\frac{du}{dy} = 2y$

$\frac{1}{2y} du = dy$

$$\int y e^{u} \cdot \frac{1}{2y} du = \int 2x+3\,dx$$

$$\frac{1}{2}\int e^{u} du = \int 2x+3\,dx$$

$$\frac{1}{2} e^{y^2+1} = \frac{2x^2}{2} + 3x + C$$

$$\frac{1}{2} e^{y^2+1} = x^2 + 3x + C$$

$$\frac{1}{2} e^{(-1)^2+1} = 0^2 + 3(0) + C$$

$$\frac{1}{2} e^{2} = C$$

$$\frac{1}{2} e^{y^2+1} = x^2 + 3x + \frac{1}{2} e^2$$

$$e^{y^2+1} = 2x^2 + 6x + e^2$$

$$\ln e^{y^2+1} = \ln(2x^2+6x+e^2)$$

$$y^2+1 = \ln(2x^2+6x+e^2)$$

$$y^2 = \ln(2x^2+6x+e^2) - 1$$

$$y = \pm\sqrt{\ln(2x^2+6x+e^2) - 1}$$

$$\boxed{y = -\sqrt{\ln(2x^2+6x+e^2) - 1}}$$

6

Unit 5 Quiz Open Ended Questions Name Key

At the beginning of this year, a power plant has a supply of 1200 tons of biofuel to burn. The function B is an increasing function and models the total amount of biofuel that the power plant has and satisfies the differential equation $\frac{dB}{dt}=\frac{3}{50}(B-200)$. B is measured in tons and t is the number of years after this year.

Find the particular solution to the differential equation $\frac{dB}{dt}=\frac{3}{50}(B-200)$, given the initial condition that $B(0)=1200$.

$$\int \frac{1}{B-200}\,dB = \int \frac{3}{50}\,dt$$

$U = B-200$

$\frac{du}{dB} = 1$

$$\int \frac{1}{u}\,du = \frac{3}{50}t + C$$

$$\ln|u| = \frac{3}{50}t + C$$

$$\ln|B-200| = \frac{3}{50}t + C$$

$$\ln|1200-200| = 0 + C$$

$$\ln(1000) = C$$

$$\ln|B-200| = \frac{3}{50}t + \ln(1000)$$

$$|B-200| = e^{3/50t + \ln(1000)}$$

$$B-200 = \pm e^{3/50t + \ln(1000)}$$

$$B = 200 \pm e^{3/50t + \ln(1000)}$$

$$\boxed{B = 200 + e^{3/50t + \ln(1000)}}$$

or

$$\boxed{B = 200 + 1000e^{3/50t}}$$