AP Calculus Unit 2 BIG ideas

EX #1) Consider the curve given by the equation $6xy = 2 + y^3$.

(a) Show that $\frac{dy}{dx} = \frac{2y}{y^2 - 2x}$.

(b) Find the coordinates of a point on the curve at which the line tangent to the curve is horizontal, or explain why no such point exists.

(c) Find the coordinates of a point on the curve at which the line tangent to the curve is vertical, or explain why no such point exists.

EX #2) Find y, y' and y'' for each equation(in I, II, III) to determine which makes the equation containing all three(called a differential equation) TRUE.

Of the following, which are solutions to the differential equation $y'' - 6y' + 8y = 0$?

I. $y = 2\sin(4x)$
II. $y = 3e^{2x}$
III. $y = Ce^{4x}$, where C is a constant.

(A) I only

(B) II only

(C) III only

(D) II and III only

EX #3) If $\lim_{x \to a} f(x) = f(a)$, then which of the following statements about f must be true?

(A) f is continuous at $x = a$.

(B) f is differentiable at $x = a$.

(C) For all values of x, $f(x) = f(a)$.

(D) The line $y = f(a)$ is tangent to the graph of f at $x = a$.

(E) The line $x = a$ is a vertical asymptote of the graph of f.

EX #4)

$$f(x) = \begin{cases} 3x - 2 & \text{if } x < 1 \\ \ln(3x - 2) & \text{if } x \geq 1 \end{cases}$$

Let f be the function defined above. Which of the following statements about f are true?

I. $\lim_{x \to 1^-} f(x) = \lim_{x \to 1^+} f(x)$

II. $\lim_{x \to 1^-} f'(x) = \lim_{x \to 1^+} f'(x)$

III. f is differentiable at $x = 1$.

(A) None

(B) I only

(C) II only

(D) II and III only

(E) I, II, and III

EX #5) USE THE GRAPHING CALCULATOR

Let f be the function defined by $f(x) = \ln(x^2 + 1)$, and let g be the function defined by $g(x) = x^5 + x^3$. The line tangent to the graph of f at $x = 2$ is parallel to the line tangent to the graph of g at $x = a$, where a is a positive constant. What is the value of a ?

(A) 0.246 (B) 0.430 (C) 0.447 (D) 0.790