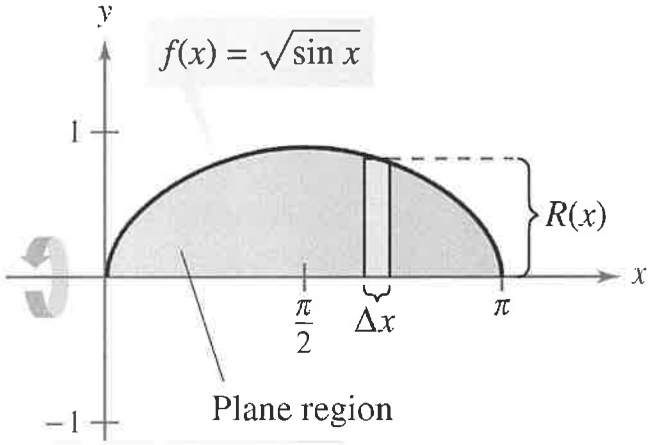
1a) Find the volume of the solid formed by revolving the region bounded by the graph of $f(x) = \sqrt{\sin x}$ and the x-axis ($0 \le x \le \pi$) about the x-axis.



$$V = \pi \int_a^b [R(x)]^2\, dx$$

$$= \pi \int_0^{\pi} \left(\sqrt{\sin x}\right)^2 dx$$

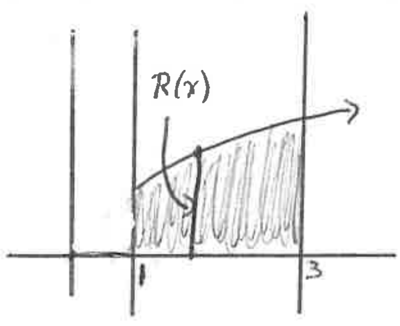
$$= \pi \int_0^{\pi} \sin x\, dx$$

$$= \pi \Big[-\cos x \Big]_0^{\pi}$$

$$= \pi(1 + 1)$$

$$= 2\pi$$

1b) Find the volume of the solid formed by revolving the region bounded by the graphs $y = \sqrt{x}$, $y = 0$, $x=1$, $x=3$ about the x-axis.



$V = \pi \cdot \int_1^3 [R(x)]^2\, dx$

$V = \pi \cdot \int_1^3 (\sqrt{x} - 0)^2\, dx$

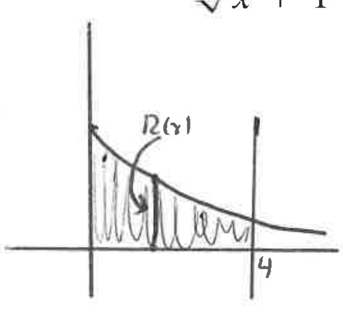
$V = \pi \cdot \int_1^3 x\, dx$

$V = \pi \cdot \left[\frac{x^2}{2} \Big|_1^3 \right]$ $V = \boxed{4\pi}$

$V = \pi \cdot \left[\frac{9}{2} - \frac{1}{2} \right]$

1c) Find the volume of the solid formed by revolving the region bounded by

the graphs $y = \dfrac{1}{\sqrt{x+1}}$, $y = 0$, $x = 0$, $x = 4$ about the x-axis.



$V = \pi \int_0^4 [R(x)]^2\, dx$

$V = \pi \int_0^4 \left[\frac{1}{\sqrt{x+1}} - 0 \right]^2 dx$

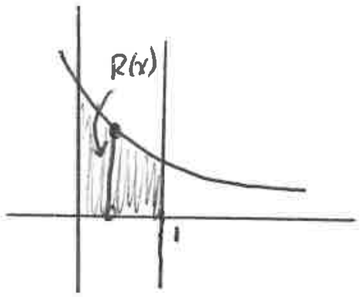
$V = \pi \int_0^4 \frac{1}{x+1}\, dx$ $u = x+1$, $\frac{du}{dx} = 1$ $= \int \frac{1}{u}\, du = \ln|u|$
$= \ln|x+1|$

$V = \pi \left[\ln|x+1| \Big|_0^4 \right]$

$V = \pi \cdot [\ln 5 - \ln 1]$

$V = \boxed{\pi \cdot \ln 5}$

1d) Find the volume of the solid formed by revolving the region bounded by the graphs $y = e^{-x}$, $y = 0$, $x = 0$, $x = 1$ about the x-axis.



$V = \pi \int_0^1 [R(x)]^2 dx$

$V = \pi \int_0^1 [e^{-x} - 0]^2 dx$

$V = \pi \int_0^1 e^{-2x} dx$ $\quad u = -2x$, $\frac{du}{dx} = -2$

$V = \pi \int_0^{-2} e^u \cdot -\frac{1}{2} du$

$V = -\frac{\pi}{2} \int_0^{-2} e^u du$

$V = -\frac{\pi}{2} [e^u |_0^{-2}]$

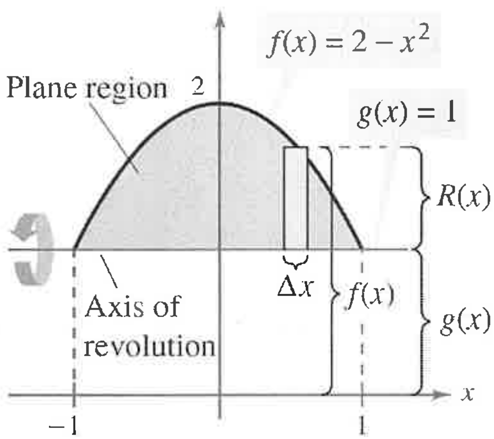
$V = -\frac{\pi}{2} [e^{-2} - e^0]$

$V = \boxed{-\frac{\pi}{2} [e^{-2} - 1]}$

or

$V = \boxed{\frac{\pi}{2} [1 - e^{-2}]}$

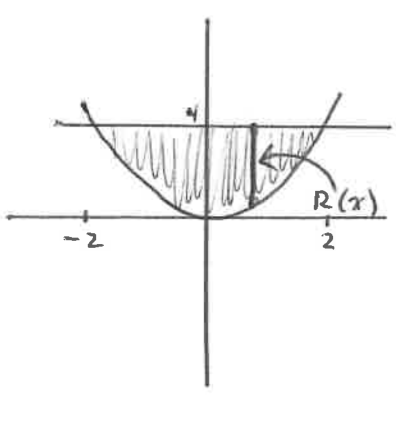
2a) Find the volume of the solid formed by revolving the region bounded by $f(x) = 2 - x^2$ and $g(x) = 1$ about the line $y = 1$



$$R(x) = f(x) - g(x)$$
$$= (2 - x^2) - 1$$
$$= 1 - x^2$$

$$V = \pi \int_a^b [R(x)]^2 dx$$
$$= \pi \int_{-1}^{1} (1 - x^2)^2 dx$$
$$= \pi \int_{-1}^{1} (1 - 2x^2 + x^4) dx$$
$$= \pi \left[x - \frac{2x^3}{3} + \frac{x^5}{5} \right]_{-1}^{1}$$
$$= \frac{16\pi}{15}$$

2b) Find the volume of the solid formed by revolving the region bounded by the graphs $y = x^2$, $y = 4$ about the line $y = 4$.



$V = \pi \int_{-2}^{2} [R(x)]^2 dx$

$V = \pi \int_{-2}^{2} [4 - x^2]^2 dx$

$V = \pi \int_{-2}^{2} [16 - 8x^2 + x^4] dx$

$V = \pi \left[\left(16x - \frac{8}{3}x^3 + \frac{x^5}{5}\right) \Big|_{-2}^{2} \right]$

$V = \pi \left[\frac{256}{15} - \left(-\frac{256}{15}\right) \right]$

$V = \boxed{\frac{512\pi}{15}}$

3a) Find the volume of the solid formed by revolving the region bounded by the graphs $y = 3(2 - x)$, $y = 0$, $x = 0$ about the line $x = 0$.

$$y = 6 - 3x \Rightarrow x = \frac{1}{3}(6 - y)$$

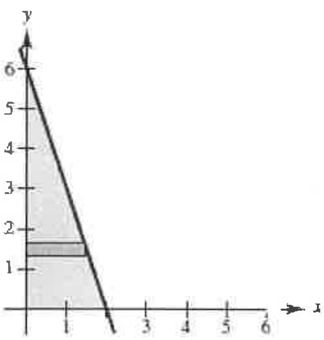
$$V = \pi\int_0^6 \left[\frac{1}{3}(6 - y)\right]^2 dy$$

$$= \frac{\pi}{9}\int_0^6 \left[36 - 12y + y^2\right] dy$$

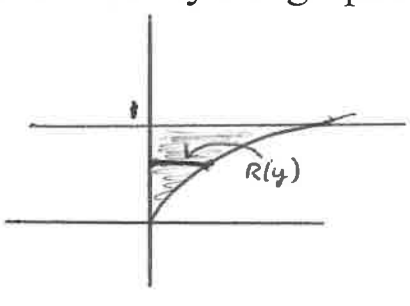
$$= \frac{\pi}{9}\left[36y - 6y^2 + \frac{y^3}{3}\right]_0^6$$

$$= \frac{\pi}{9}\left[216 - 216 + \frac{216}{3}\right]$$

$$= 8\pi = \frac{1}{3}\pi r^2 h, \text{ Volume of cone}$$



3b) Find the volume of the solid formed by revolving the first quadrant region bounded by the graphs $y = x^{2/3}$, $x = 0$, $y = 1$ about the line $x = 0$.



$$V = \pi \int_0^1 [R(y)]^2 dy$$

$$V = \pi \int_0^1 [y^{3/2} - 0]^2 dy$$

$$V = \pi \int_0^1 y^3 dy$$

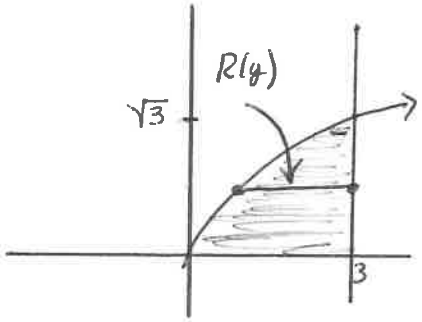
$$V = \pi \left[\frac{y^4}{4}\Big|_0^1\right]$$

If $y = x^{2/3}$ then $y^{3/2} = x$

$$V = \pi\left[\tfrac{1}{4} - 0\right]$$

$$V = \boxed{\frac{\pi}{4}}$$

3c) Find the volume of the solid formed by revolving the region bounded by the graphs $y = \sqrt{x}$, $y = 0$, $x = 3$ about the line $x = 3$.



$$V = \pi \int_0^{\sqrt{3}} [R(y)]^2 dy$$

If $y = \sqrt{x}$ then $y^2 = x$

$$V = \pi \int_0^{\sqrt{3}} [3 - y^2]^2 dy$$

$$V = \pi \int_0^{\sqrt{3}} [9 - 6y^2 + y^4] dy$$

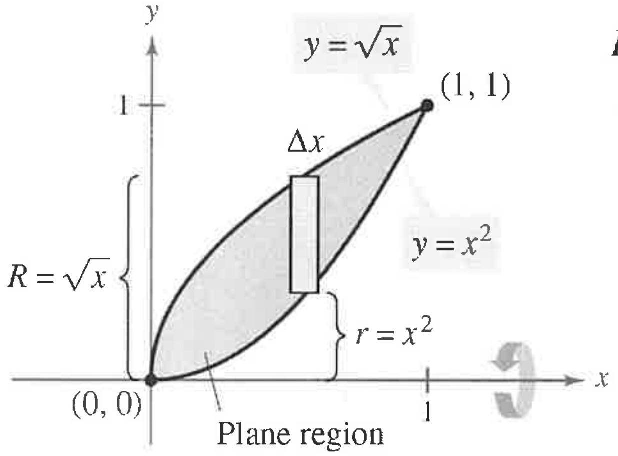
$$V = \pi \left[\left(9y - 2y^3 + \frac{y^5}{5}\right)\Big|_0^{\sqrt{3}}\right]$$

$$V = \pi \left[9\sqrt{3} - 2(\sqrt{3})^3 + \frac{(\sqrt{3})^5}{5} - (0 - 0 + 0)\right]$$

$$V = \boxed{\pi\left[9\sqrt{3} - 2\sqrt{27} + \tfrac{1}{5}\sqrt{243}\right] \text{ or } \frac{24\sqrt{3}\,\pi}{5}}$$

"if you simplify radicals"

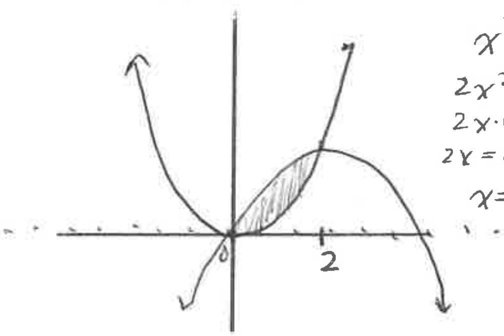
4a) Find the volume of the solid formed by revolving the region bounded by the graphs $y = \sqrt{x}$ and $y = x^2$ about the x-axis



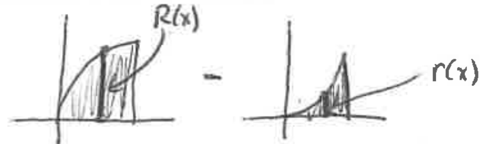
$R(x) = \sqrt{x}$
$r(x) = x^2$

$$V = \pi \int_a^b \left([R(x)]^2 - [r(x)]^2\right) dx$$
$$= \pi \int_0^1 \left[(\sqrt{x})^2 - (x^2)^2\right] dx$$
$$= \pi \int_0^1 (x - x^4)\, dx$$
$$= \pi \left[\frac{x^2}{2} - \frac{x^5}{5}\right]_0^1$$
$$= \frac{3\pi}{10}$$

4b) Find the volume of the solid formed by revolving the region bounded by the graphs $y = x^2$, $y = 4x - x^2$ about the x-axis.



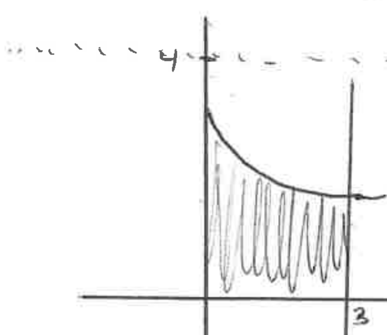
$x^2 = 4x - x^2$
$2x^2 - 4x = 0$
$2x \cdot (x-2) = 0$
$2x = 0 \quad x - 2 = 0$
$x = 0$ or $x = 2$

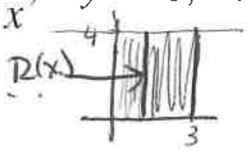


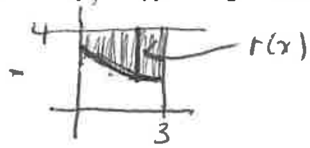
$V = \pi \int_0^2 [R(x)]^2 dx - \pi \int_0^2 [r(x)]^2 dx$
$V = \pi \int_0^2 (4x - x^2)^2 dx - \pi \int_0^2 [x^2]^2 dx$
$V = \pi \int_0^2 (4x - x^2)^2 - (x^2)^2 dx$
$V = \pi \int_0^2 16x^2 - 8x^3 dx$

$V = \pi \left[\frac{16}{3}x^3 - 2x^4\right]_0^2$
$V = \pi \left[\left(\frac{128}{3} - 32\right) - (0 - 0)\right]$
$V = \pi \cdot \frac{32}{3}$
$V = \boxed{\frac{32\pi}{3}}$

4c) Find the volume of the solid formed by revolving the region bounded by the graphs $y = \dfrac{3}{1+x}$, $y = 0$, $x = 0$, $x = 3$ about the line $y = 4$.



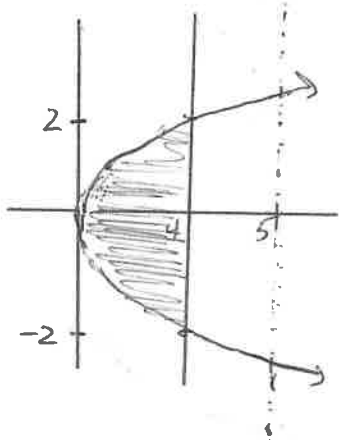


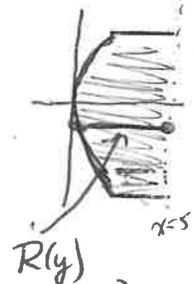


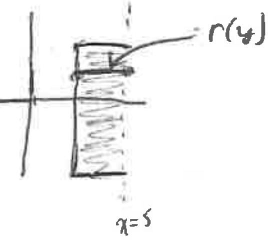
$V = \pi \int_0^3 [R(x)]^2 dx - \pi \int_0^3 [r(x)]^2 dx$
$V = \pi \int_0^3 (4 - 0)^2 dx - \pi \int_0^3 \left(4 - \frac{3}{1+x}\right)^2 dx$
$V = \pi \int_0^3 16 - \left(4 - \frac{3}{1+x}\right)^2 dx \quad \Rightarrow$ G.C. $\approx \boxed{83.318}$

4d) Find the volume of the solid formed by revolving the region bounded by

the graphs $x = y^2$, $x = 4$ about the line $x = 5$.







$$V = \pi \int_{-2}^{2} (5-y^2)^2 dy - \pi \int_{-2}^{2} (5-4)^2 dy$$

$$V = \pi \int_{-2}^{2} (5-y^2)^2 - 1^2 dy$$

$$V = \pi \int_{-2}^{2} 24 - 10y^2 + y^4 dy$$

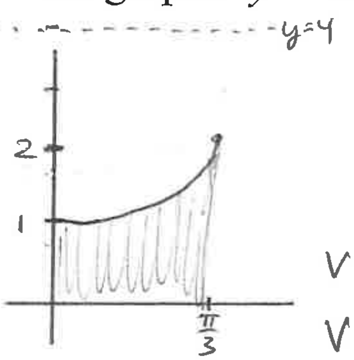
$$V = \pi \left[24y - \frac{10}{3}y^3 + \frac{1}{5}y^5 \Big|_{-2}^{2} \right]$$

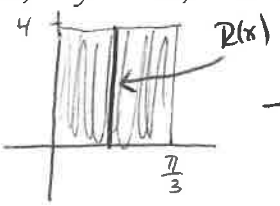
$$V = \pi \left[\frac{416}{15} - \left(-\frac{416}{15}\right) \right]$$

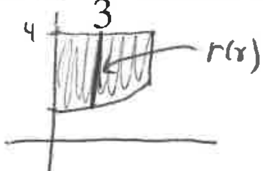
$$V = \boxed{\frac{832}{15}\pi}$$

4e) Find the volume of the solid formed by revolving the region bounded by

the graphs $y = \sec x$, $y = 0$, $0 \le x \le \dfrac{\pi}{3}$ about the line $y = 4$.







$$V = \pi \int_0^{\frac{\pi}{3}} (R(x))^2 dx - \pi \int_0^{\frac{\pi}{3}} (r(x))^2 dx$$

$$V = \pi \int_0^{\frac{\pi}{3}} (4-0)^2 dx - \pi \int_0^{\frac{\pi}{3}} (4-\sec x)^2 dx$$

$$V = \pi \int_0^{\frac{\pi}{3}} 16 - (4-\sec x)^2 dx$$

$$V = \pi \int_0^{\frac{\pi}{3}} 8\sec x - \sec^2 x \, dx$$

$$V = \pi \left[(8\ln|\sec x + \tan x| - \tan x) \Big|_0^{\frac{\pi}{3}} \right]$$

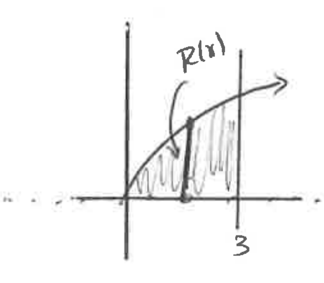
$$V = \pi \left[(8\ln|2+\sqrt{3}| - \sqrt{3}) - (8\ln 1) \right]$$

$$\boxed{V = \pi(8\ln|2+\sqrt{3}| - \sqrt{3})}$$

4) AP MULTIPLE CHOICE EXAMPLES

1) The region enclosed by the x-axis, the line $x = 3$, and the curve $y = \sqrt{x}$ is rotated about the x-axis. What is the volume of the solid generated?

(A) 3π (B) $2\sqrt{3}\pi$ (C) $\boxed{\frac{9}{2}\pi}$ (D) 9π (E) $\frac{36\sqrt{3}}{5}\pi$



$$V = \pi \int_0^3 [R(x)]^2 dx$$

$$V = \pi \int_0^3 (\sqrt{x})^2 dx$$

$$V = \pi \int_0^3 x \, dx$$

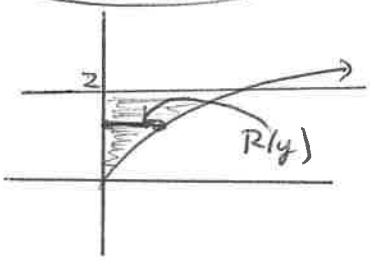
$$V = \pi \left[\frac{x^2}{2} \Big|_0^3 \right]$$

$$V = \pi \left[\frac{9}{2} - 0 \right]$$

$$\frac{9\pi}{2}$$

2) If the region enclosed by the y-axis, the line $y = 2$, and the curve $y = \sqrt{x}$ is revolved about the y-axis, the volume of the solid generated is

(A) $\frac{32\pi}{5}$ (B) $\frac{16\pi}{3}$ (C) $\frac{16\pi}{5}$ (D) $\frac{8\pi}{3}$ (E) π



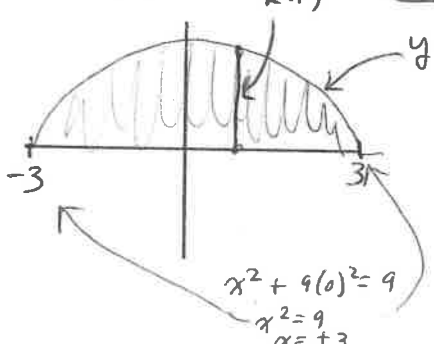
If $y = \sqrt{x}$

$y^2 = x$

$$V = \pi \int_0^2 [R(y)]^2 \, dy$$
$$V = \pi \int_0^2 [y^2]^2 \, dy$$
$$V = \pi \int_0^2 y^4 \, dy$$
$$V = \pi \left[\frac{y^5}{5} \Big|_0^2 \right]$$
$$\pi \left[\frac{32}{5} - 0 \right]$$
$$\frac{32\pi}{5}$$

3) The volume of the solid obtained by revolving the region enclosed by the ellipse $x^2 + 9y^2 = 9$ about the x-axis is

(A) 2π (B) 4π (C) 6π (D) 9π (E) 12π

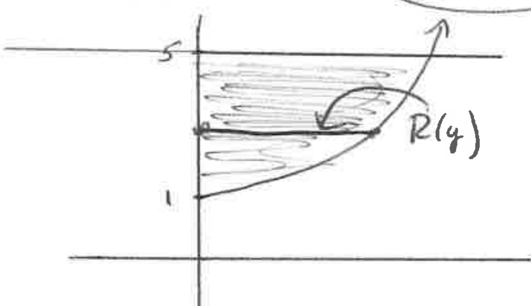


$$y = \pm\sqrt{\frac{9-x^2}{9}}$$

$$V = \pi \int_{-3}^{3} [R(x)]^2 \, dx$$
$$V = \pi \int_{-3}^{3} \left(\sqrt{\frac{9-x^2}{9}}\right)^2 dx$$
$$V = \pi \int_{-3}^{3} 1 - \frac{1}{9}x^2 \, dx$$
$$V = \pi \left[\left(x - \frac{1}{27}x^3\right)\Big|_{-3}^{3} \right]$$
$$V = \pi \left[(3-1) - (-3-(-1)) \right]$$
$$V = \pi \left[2 - (-2) \right]$$
$$4\pi$$

4) The region R in the first quadrant is enclosed by the lines $x = 0$ and $y = 5$ and the graph of $y = x^2 + 1$. The volume of the solid generated when R is revolved about the y-axis is

(A) 6π (B) 8π (C) $\frac{34\pi}{3}$ (D) 16π (E) $\frac{544\pi}{15}$



If $y = x^2 + 1$

$y - 1 = x^2$

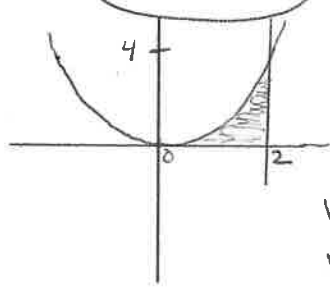
$\pm\sqrt{y-1} = x$

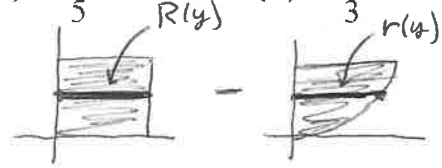
$+\sqrt{y-1} = x$

$$V = \pi \int_1^5 (R(y))^2 \, dy$$
$$V = \pi \int_1^5 \left(\sqrt{y-1} - 0\right)^2 dy$$
$$V = \pi \int_1^5 y - 1 \, dy$$
$$V = \pi \left[\left(\frac{y^2}{2} - y\right)\Big|_1^5 \right]$$
$$V = \pi \left[\left(\frac{25}{2} - 5\right) - \left(\frac{1}{2} - 1\right) \right]$$
$$V = \pi \left[\frac{15}{2} - \left(-\frac{1}{2}\right) \right]$$
$$V = \pi \left[\frac{16}{2} \right]$$
$$V = 8\pi$$

5) The region enclosed by the graph of $y = x^2$, the line $x = 2$, and the x-axis is revolved about the y-axis. The volume of the solid generated is

(A) 8π (B) $\frac{32}{5}\pi$ (C) $\frac{16}{3}\pi$ (D) 4π (E) $\frac{8}{3}\pi$





$V = \pi\int_0^4 [R(y)]^2 dy - \pi\int_0^4 [r(y)]^2 dy$

$V = \pi\int_0^4 (2-0)^2 dy - \pi\int_0^4 (\sqrt{y}-0)^2 dy$

$\to V = \pi\int_0^4 4 - y \, dy$

$V = \pi\left[\left(4y - \frac{y^2}{2}\right)\Big|_0^4\right]$

$V = \pi[(16-8)-(0-0)]$

$V = 8\pi$

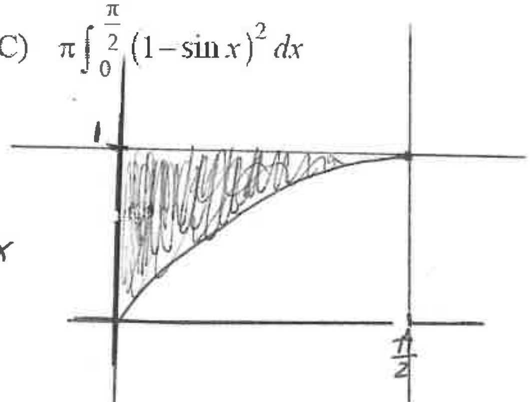
if $y = x^2$

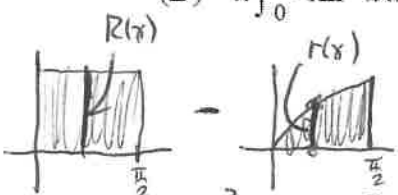
$\sqrt{y} = x$

6) Let R be the region between the graphs of $y = 1$ and $y = \sin x$ from $x = 0$ to $x = \frac{\pi}{2}$. The volume of the solid obtained by revolving R about the x-axis is given by

(A) $2\pi\int_0^{\frac{\pi}{2}} x\sin x \, dx$ (B) $2\pi\int_0^{\frac{\pi}{2}} x\cos x \, dx$ (C) $\pi\int_0^{\frac{\pi}{2}} (1-\sin x)^2 dx$

(D) $\pi\int_0^{\frac{\pi}{2}} \sin^2 x \, dx$ (E) $\pi\int_0^{\frac{\pi}{2}} (1-\sin^2 x) dx$





$V = \pi\int_0^{\pi/2} (R(x))^2 dx - \pi\int_0^{\pi/2} (r(x))^2 dx$

$\to V = \pi\int_0^{\pi/2} (1-0)^2 dx - \pi\int_0^{\pi/2} (\sin x - 0)^2 dx$

$\boxed{V = \pi\int_0^{\pi/2} 1 - \sin^2 x \, dx}$

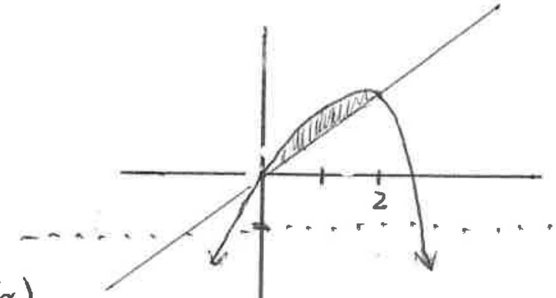
7) The region R is enclosed by the graph of $y = 3x - x^2$ and the line $y = x$. If the region R is rotated about the line $y = -1$, the volume of the solid that is generated is represented by which of the following integrals?

(A) $\pi\int_0^2 \left[3x - x^2 - x + 1\right]^2 dx$

(B) $\pi\int_0^2 \left[\left(3x - x^2 + 1\right)^2 - (x+1)^2\right] dx$

(C) $\pi\int_0^2 \left[\left(3x - x^2 + 1\right) - (x+1)\right]^2 dx$

(D) $\pi\int_0^2 \left[\left(3x - x^2 - 1\right)^2 - (x-1)^2\right] dx$



$V = \pi\int_0^2 (R(x))^2 dx - \pi\int_0^2 (r(x))^2 dx$

$V = \pi\int_0^2 (3x - x^2 - (-1))^2 dx - \pi\int_0^2 (x - (-1))^2 dx$

$V = \pi\int_0^2 (3x - x^2 + 1)^2 dx - \pi\int_0^2 (x+1)^2 dx$